II. *On the Attraction of the Himalaya Mountains, and of the elevated regions beyond them, upon the Plumb-line in India. By the Venerable* JOHN HENRY PRATT, *M.A., Archdeacon of Calcutta. Communicated by the Rev.* J. CHALLIS, *M.A., F.R.S. &c.*

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1. IT is now well known that the attraction of the Himalaya Mountains, and of the elevated regions lying beyond them, has a sensible influence upon the plumb-line in North India. This circumstance has been brought to light during the progress of the great trigonometrical survey of that country. It has been found by triangulation that the difference of latitude between the two extreme stations of the northern division of the arc, that is, between Kalianpur and Kaliana, is 5° 23′ 42″·294, whereas astronomical observations show a difference of 5° 23″ 37″·058, which is 5″·236* less than the former.

2. That the geodetic operations are not in fault appears from this; that two bases, about seven miles long, at the extremities of the arc having been measured with the utmost care, and also the length of the northern base having been computed from the measured length of the southern one, through a chain of triangles stretching along the whole arc, about 370 miles in extent, the difference between the measured and the computed lengths of the northern base was only 0·6 of a foot, an error which would produce, even if wholly lying in the meridian, a difference of latitude no greater than 0″·006.

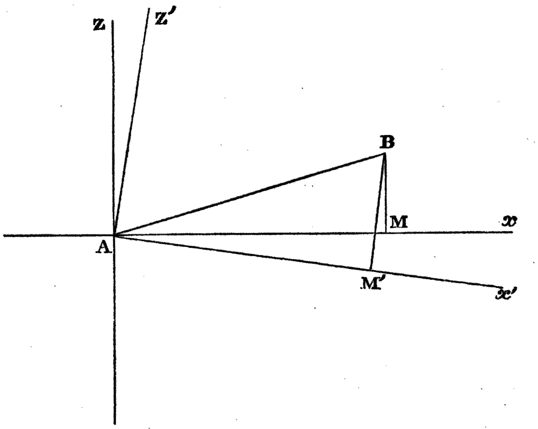
3. The difference 5″·236 must therefore be attributed to some other cause than error in the geodetic operations. A very probable cause is the attraction of the superficial matter which lies in such abundance on the north of the Indian arc. This disturbing cause acts in the right direction; for the tendency of the mountain mass must be to draw the lead of the plumb-line at the northern extremity of the arc more to the north than at the southern extremity, which is further removed from the attracting mass. Hence the effect of the attraction will be to lessen the difference of latitude, which is the effect observed. Whether this cause will account for the error in the difference of latitude in *quantity*, as well as in direction, remains to be considered, and is the question I propose to discuss in the present paper.

4. But if mountain attraction have any sensible influence at the stations on the arc, how is it that the geodetic operations are not affected by it? How is it that such a remarkable degree of exactness between the measured and computed lengths of the

* This is the difference as stated by Colonel EVEREST in his work on the Measurement of the Meridional Arc of India, published in 1847. See p. clxxviii.

northern base attests, it would seem, to the non-existence of any external disturbing cause? For in observing the altitude or depression of one station in the triangulation as seen from another, the error on the plumb-line must come into the calculation. The answer is, that these small errors occur in the calculation of the horizontal arc in very small terms not higher than the second order; whereas in the expression for the inclination of the two verticals at the extremities of the arc they occur in terms of the first order. This I will further illustrate.

Fig. 1.



5. Suppose the arc divided into n equal portions: and let $\nu_0\nu_1\nu_2.....\nu_n$ be the deflections of the plumb-line at the $n+1$ stations thus chosen. Let A be one of these stations, and B the next towards the south; Az, Ax vertical and horizontal lines through A on the supposition that there is no mountain attraction; Az', Ax' the vertical and horizontal lines as affected by attraction. Draw BM and BM$'$ perpendicular to Ax and Ax': let AM$=a$, BM$=h$, $\angle z\text{A}z'=\nu$, $\angle$BAM$=\alpha$. Then AM is the true horizontal distance between A and B, and AM$'$ the calculated horizontal distance. Hence the calculation makes this portion of the arc too short by

$$\text{AM}-\text{AM}'=\text{AM}\left(1-\frac{\cos(\alpha+\nu)}{\cos\alpha}\right)=a(\tan\alpha\,.\sin\nu+1-\cos\nu)=h.\nu+\frac{1}{2}a.\nu^2,$$

neglecting the cube and higher powers of ν.

Hence the whole arc is made too short by

$$h_0\nu_0+h_1\nu_1+h_2\nu_2+\ldots.+h_n\nu_n+\frac{a}{2}(\nu_0^2+\nu_1^2+\nu_2^2+\ldots.+\nu_n^2),$$

$h_0h_1h_2...h_n$ being the heights of the various stations of observation above the true horizontal line. When the Station B is below A then h is negative. These heights are all extremely small compared with a, as the arc lies through a comparatively flat country. Hence the expression for the error in the length of the arc is made up, as I said, of small terms of no higher order than the second; whereas the error in the difference of latitude $(=\nu_n-\nu_0)$ has terms of the first order.

6. That this expression for the shortening of the arc is a minute quantity utterly inappreciable, may easily be shown by taking an extreme case. The quantities $h_0h_1h_2...h_n$ are some of them positive and some of them negative, in such a manner that their algebraical sum equals the difference of height of Kalianpur and Kaliana above the level of the sea. From Colonel EVEREST's work on the Indian Arc (published in 1847) I gather, that between Kalianpur and Kaliana there are forty-seven principal stations, or, including the two terminal ones, forty-nine: and the Survey

shows that in passing from north to south there are twenty-five elevations of one station above the level of the preceding one, amounting in all to 3901·1 feet; and twenty-three depressions, amounting to 2965·2 feet (see pp. 269–273). The difference between these =935·9 feet, which is the height assigned to Kalianpur above Kaliana. I will take these, then, as the values of $h_0 h_1 h_2 \ldots h_n$ in my present example; so that the sum of the positive quantities among $h_0 h_1 h_2 \ldots = 3901 \cdot 1$ feet, and the sum of the negative $= 2965 \cdot 2$, and $n = 48$. Now ν_n is the greatest and ν_0 is the least of the quantities $\nu_n \ldots \nu_0$. Hence it follows, that

$$h_0\nu_0 + h_1\nu_1 + \ldots + h_n\nu_n \text{ is less than } 3901 \cdot 1\nu_n - 2 \cdot 965 . 2\nu_0,$$

and therefore, much more, less than $3901 \cdot 1\nu_n$ feet.

Now by the Survey $\nu_n - \nu_0 = 5'' \cdot 236$, or in arcs $= 0 \cdot 000025$;

$$\therefore \quad h_0\nu_0 + h_1\nu_1 + \ldots + h_n\nu_n \text{ is less than } 3901 \cdot 1 \times 0 \cdot 000025 \text{ or } 0 \cdot 097527 \text{ foot.}$$

If in this extreme case of supposing the attraction to equal its greatest value at more than half of the stations, and that at stations where its effect would be greatest, the result is so insignificant, what must it be in the actual case*? The same may be shown with respect to the other term in the expression for the shortening of the arc, viz. $\frac{1}{2} a(\nu_0^2 + \nu_1^2 + \ldots \nu_n^2)$. This quantity is less than $\frac{n+1}{2} a . \nu_n^2$; or, if we reckon the distance between Kaliana and Kalianpur to be 370 miles, and therefore $n . a = 370 \times 1760 \times 3$ feet and $n = 48$, this quantity is less than 0·008 of a foot, which is utterly inappreciable. Hence mountain attraction may have a sensible value at the stations on the arc, and yet not affect the *geodetic* calculations in the slightest appreciable degree†.

7. I can see no ground, therefore, whatever for the process of dispersion which Colonel Everest describes at page clxx of the Introduction to his work, by which he distributes the error 5″·236 among the triangles. It appears to me to be unquestionable that the geodetic operations are in no way sensibly affected by mountain attraction, and therefore need no correction whatever on that account. It is the *astronomical* operation of observing the difference of latitude which requires the correction. That it is here that the correction must be applied appears again in attempting to determine the azimuths of the arc at seven stations *astronomically* (see p. xlii). It is only when the plumb-line is brought into use to determine the vertical angles of stars that the effect of attraction becomes sensible; and never in the geodetic calculations, where only horizontal angles or extremely minute vertical angles (viz. the elevations or depressions of $h_0 h_1 h_2 \ldots$) are observed.

8. The importance of accounting satisfactorily for the difference between the geodetic

* If the triangulation be carried into elevated regions some of the values of $h_0 h_1 h_2 \ldots$ will be large; and therefore the conclusion in the text will not in that case stand.

† In this paper I show that the difference caused by attraction in the latitudes of the extremities of the northern division of the arc, viz. Kaliana and Kalianpur, amounts to 15″·885, which is more than three times the angle 5″·236. But the conclusion arrived at in art. 6. is still true.

and astronomical results appears from the effect it must have upon the determination of the earth's ellipticity; an effect such, that unless this quantity be fully accounted for, it must render the great Indian Survey comparatively useless in the delicate problem of the Figure of the Earth, however valuable it may be for the purposes of mapping the vast continent of Hindostan.

9. The effect of a small error in the difference of latitude upon the determination of the ellipticity may be calculated as follows:—

Let ε be the ellipticity, a quantity known not to differ much from $\frac{1}{300}$; λ the amplitude of the arc; μ the latitude of the middle point of the arc. Then by the usual formula

$$\frac{\text{length of arc}}{\text{equatorial radius}}=\lambda-\frac{1}{2}\varepsilon(\lambda+3\sin\lambda\cos 2\mu).$$

But $\sin\lambda=\lambda-\frac{1}{6}\lambda^3+..=\lambda\left(1-\frac{1}{6}\lambda^2+..\right)$; $\lambda=5^\circ\ 23'\ 37''$ for the arc between Kalianpur and Kaliana $=0{\cdot}094$ in parts of the radius,

$$\therefore\quad \frac{1}{6}\lambda^2=0{\cdot}00147.$$

Hence by putting λ instead of $\sin\lambda$ in the above formula, we shall be omitting a quantity of the order $\frac{1}{2}\varepsilon\times 0{\cdot}00441\cos 2\mu$, which is utterly insignificant,

$$\therefore\quad \frac{\text{length of arc}}{\text{equatorial radius}}=\lambda\left(1-\frac{1}{2}\varepsilon\right)\left(1-\frac{3}{2}\varepsilon\cos 2\mu\right).$$

In the same way if L be the amplitude and M the latitude of the middle point of another arc,

$$\frac{\text{length of arc L}}{\text{equatorial radius}}=\mathrm{L}\left(1-\frac{1}{2}\varepsilon\right)\left(1-\frac{3}{2}\varepsilon\cos 2\mathrm{M}\right),$$

$$\therefore\quad \frac{\text{length of arc }\lambda}{\text{length of arc L}}=\frac{\lambda}{\mathrm{L}}\left\{1-\frac{3}{2}\varepsilon(\cos 2\mu-\cos 2\mathrm{M})\right\}.$$

Suppose the observed values of λ and μ are subject to small errors owing to mountain attraction; to find the effect on ε we must differentiate this expression, supposing the angles λ, μ and ε variable and the other quantities constant,

$$\therefore\quad 0=d\lambda\left\{1-\frac{3}{2}\varepsilon(\cos 2\mu-\cos 2\mathrm{M})\right\}$$

$$+3\lambda\varepsilon\sin 2\mu\,.\,d\mu-\frac{3}{2}\lambda(\cos 2\mu-\cos 2\mathrm{M})d\varepsilon,$$

$$\therefore\quad d\varepsilon=\frac{d\lambda}{\lambda}\,\frac{2}{3(\cos 2\mu-\cos 2\mathrm{M})},$$

neglecting extremely small quantities of the higher order.

Now in the case before us,

$$\lambda=\text{ latitude of Kaliana }-\text{ latitude of Kalianpur},$$
$$=29^\circ\ 30'\ 48''-24^\circ\ 7'\ 11''=5^\circ\ 23'\ 37'',$$

μ= half the sum of these latitudes,
$=26° 49'$; $\cos 2\mu=0{\cdot}59295$.

Suppose $d\lambda=1''$ only; then

$$d\varepsilon=\frac{1''}{5°\ 23'\ 38''}\ \frac{2}{3(0{\cdot}59295-\cos 2M)}$$

$$=\frac{1}{58254}\ \frac{2}{0{\cdot}59295-\cos 2M}.$$

This will be smallest when 2M is chosen as nearly 180° as possible. The great arc lately measured near North Cape is the one which will best meet this condition. Put therefore $M=70°$, $\cos 2M=-0{\cdot}76604$, and

$$\therefore\quad d\varepsilon=\frac{1}{58254}\ \frac{2}{1{\cdot}35899}=\frac{1}{39585}$$

$$=\frac{\varepsilon}{132},\ \text{if we put } \frac{1}{300} \text{ for } \varepsilon.$$

Hence for an error of $5''{\cdot}236$ in defect in the amplitude, the effect on the ellipticity will be to diminish it by $\frac{5{\cdot}236}{132}\varepsilon=\frac{\varepsilon}{25}$ nearly, or by nearly $\frac{1}{25}$th part of its whole value, under the most favourable circumstances. This is sufficient to show the great importance of endeavouring to account satisfactorily for the discrepancy brought to light by the Indian Survey; and that, not by merely putting it down to mountain attraction, but by calculating that attraction by some independent means, with a view to see whether its amount actually corresponds with the observed anomaly*.

10. To dissect and actually to calculate the attraction of the masses of which the Himalayas, and the regions beyond, are composed, appears, at the very thought of it, to be an herculean undertaking next to impossible. I am fully convinced, however, that no other method will succeed. It is upon this plan that the solution of the problem is conducted in this paper. It will be seen, that by selecting a peculiar law of dissection the calculation is very greatly simplified, and made to depend entirely and solely upon a knowledge of the elevations and depressions, in fact, the general contour of the surface. This information for some part of the mass is already supplied by the maps of the Trigonometrical Survey.

11. In the following pages I propose, in the first place, to develope my method of calculation, and to deduce a formula by which the attraction can be determined with a precision corresponding to the degree of accuracy to which the contour of the surface is known.

* If the effect of mountain attraction upon the northern division of the arc be what I make it, $15''{\cdot}885$, then the ellipticity as determined from this and the Russian arc would be too small by $\frac{1}{8}\varepsilon$, if mountain attraction is neglected. The error in the ellipticity in comparing the whole arc between Kaliana and Damargida with the North Cape arc will, under the same circumstances, amount even to $\frac{1}{6}\varepsilon$. This will appear from the sequel, and is here mentioned only to illustrate the importance of the subject under consideration.

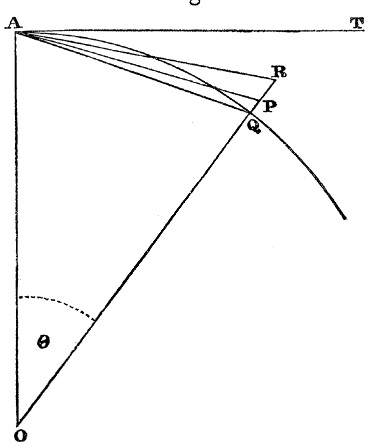
In the second place, I propose to reduce the formula to numbers, and so arrive at such an approximate value of the attraction as the data I have been able to collect will allow.

12. This approximate value is, as will be seen, larger than $5''\cdot236$, the error brought to light by the Survey. I make various suppositions with a view, if possible, to reduce my result to this, but without effect. This leads me to look in another direction for an explanation of the cause of discordance, and I arrive at a conclusion which clears up the discrepancy, confirms the calculations of this paper, and illustrates the importance of not disregarding the influence of mountain attraction.

I. *Determination of a Formula for calculating Mountain Attraction on the stations of the Indian Arc.*

13. Let O be the centre of a circle AQ, AT the tangent at A, QR a slender prism of mass M, being the prolongation of the radius through the point Q. Then if $AQ=a$, $AR=b$, $\angle QAR=\omega$, and $\angle AOQ=\theta$, the following is true:—

Fig. 2.



Lemma.—The attraction of the prism QR on the point A in the direction AT

$$=\frac{M}{ab}\cos\frac{\theta}{2}\left\{1+\tan\frac{\theta}{2}\tan\frac{\omega}{2}\right\}.$$

For let P be any point of the prism, $QP=z$, $QR=h$, $\angle PAQ=\psi$,

$\therefore$ mass of element of prism at $P=M\frac{dz}{h}$,

$$\text{attraction of this on A in direction AP}=M\frac{dz}{h}\,\frac{1}{PA^2},$$

$$\text{attraction of this on A in direction AT}=M\frac{dz}{h}\,\frac{\cos PAT}{PA^2},$$

$$\text{Cos PAT}=\cos\left(\frac{1}{2}\theta-\psi\right)$$

$$\frac{AP}{a}=\frac{\cos\frac{1}{2}\theta}{\cos\left(\frac{1}{2}\theta+\psi\right)},\quad \frac{h}{b}=\frac{\sin\omega}{\cos\frac{1}{2}\theta},$$

$$z=QP=a\frac{\sin\psi}{\cos\left(\frac{1}{2}\theta+\psi\right)}=a\left(\cos\frac{1}{2}\theta\tan\left(\frac{1}{2}\theta+\psi\right)-\sin\frac{1}{2}\theta\right)$$

$$\frac{dz}{d\psi}=a\cos\frac{1}{2}\theta\sec^2\left(\frac{1}{2}\theta+\psi\right).$$

Putting these values in the above expression, attraction of element at P on A in direction AT

$$=\frac{M}{ab \sin \omega} \cos\left(\frac{1}{2}\theta-\psi\right)d\psi;$$

∴ attraction of prism QR on A in direction AT

$$=\frac{M}{ab\sin\omega}\left\{\text{constant}-\sin\left(\frac{1}{2}\theta-\psi\right)\right\} \text{ from } \psi=0 \text{ to } \psi=\omega;$$

$$=\frac{M}{ab\sin\omega}\left\{\sin\frac{1}{2}\theta-\sin\left(\frac{1}{2}\theta-\omega\right)\right\}$$

$$=\frac{M}{ab}\cos\frac{1}{2}\theta\left\{1+\frac{1-\cos\omega}{\sin\omega}\tan\frac{1}{2}\theta\right\}$$

$$=\frac{M}{ab}\cos\frac{1}{2}\theta\left\{1+\tan\frac{1}{2}\omega\tan\frac{1}{2}\theta\right\}.$$

14. *Corollary.* The above formula will reduce itself to the following in the cases to which we have to apply it.

$$\text{Attraction of prism on A in direction AT}=\frac{M}{a^2}\cos\frac{1}{2}\theta.$$

For in these cases AQ is the circumference, and O the centre, of the earth; QR the height of any point of the surface above the sea-level at A.

Now $$\frac{a}{b}=\frac{\cos\left(\frac{1}{2}\theta+\omega\right)}{\cos\frac{1}{2}\theta}=\cos\omega-\sin\omega.\tan\frac{\theta}{2};$$

∴ attraction of prism

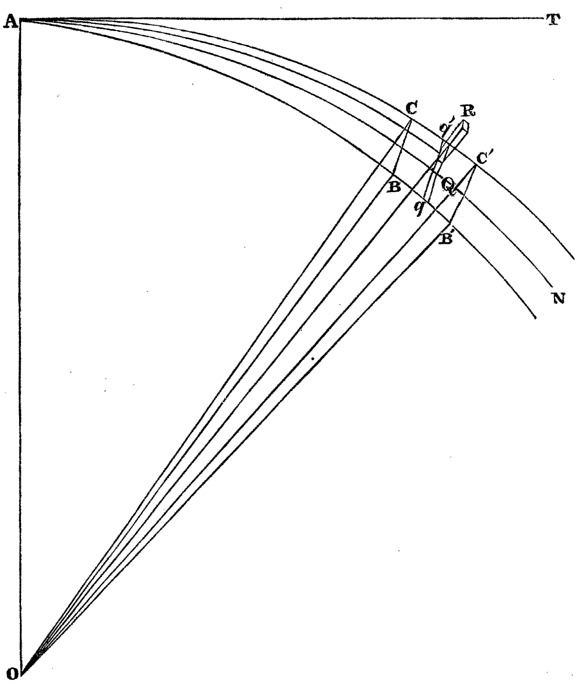
$$=\frac{M}{a^2}\cos\frac{1}{2}\theta\left\{1+\tan\frac{1}{2}\omega\tan\frac{1}{2}\theta\right\}\times\left\{\cos\omega-\sin\omega\tan\frac{1}{2}\theta\right\}$$

$$=\frac{M}{a^2}\cos\frac{1}{2}\theta\left\{1-\frac{1}{2}\omega\tan\frac{1}{2}\theta\right\},$$

neglecting the square and higher powers of ω. But ω is never greater than 2° (=0·03488 in arcs), and when it has this maximum value, θ is less than 2°; and as θ increases, ω decreases in a higher degree. Hence the second term within the brackets is of insensible importance, and the corollary as enunciated is true.

15. In order to calculate the attraction of the superficial crust of the earth upon the point A on its surface, I shall suppose a number of vertical planes to be drawn through A making any angles with each other, and thus dividing the surface through A, parallel to the sea-level, into a number of *lunes* all meeting again in a point in the antipodes of A. About A as centre suppose a number of concentric circles drawn on this surface; the law of the distances of these circles will be determined hereafter. In this way the whole surface will be divided into a number of four-sided *compartments,* two of the sides in every compartment converging to A, and the other two being parts of circles concentric in A.

Fig. 3.



Let ABB′ and ACC′ be parts of two of the great circles forming these lunes; BCC′B′ one of the four-sided compartments. Let $\angle BAC=\beta$, $\angle AOB=\alpha$, $\angle BOB'=\phi$; Q an element of the compartment; qQq' parallel to BC; $\angle QAN=\psi$, AN being a great circle bisecting the angle β, and AT a tangent to AN; $AO=r$, $\angle AOQ=\theta$;

$\therefore$ distance of Q from $AO=r\sin\theta$;

and $r\sin\theta\, d\psi$, and $rd\theta$ are the sides of the element Q. Let k' be the height of R, the earth's surface, above Q; ρ the mean density of the superficial matter of the earth;

$\therefore$ mass of prism $QR=\rho k' r^2 \sin\theta\, d\theta\, d\psi$;

also chord $AQ=2r\sin\frac{1}{2}\theta$.

Hence by the corollary in Art. 14,

attraction of prism QR on A along the tangent to AQ

$$=\rho\frac{k'r^2\sin\theta\, d\theta\, d\psi}{4r^2\sin^2\frac{1}{2}\theta}\cos\frac{1}{2}\theta;$$

$\therefore$ attraction of prism QR on A along AT

$$=\rho\frac{k'r^2\sin\theta\, d\theta\, d\psi}{4r^2\sin^2\frac{1}{2}\theta}\cos\frac{1}{2}\theta\cos\psi.$$

Integrating with respect to ψ, from $\psi=-\frac{1}{2}\beta$ to $\frac{1}{2}\beta$, attraction on A of the mass standing on qq' in direction AT

$$=\frac{\rho\sin\theta\cos\frac{1}{2}\theta\, d\theta}{4\sin^2\frac{1}{2}\theta}\int_{-\frac{1}{2}\beta}^{\frac{1}{2}\beta}k'\cos\psi d\psi=\frac{\rho\sin\theta\cos\frac{1}{2}\theta\, d\theta}{4\sin^2\frac{1}{2}\theta}\cdot 2\sin\frac{1}{2}\beta\,.\,k \text{ very nearly,}$$

k being the average value of k' on the arc qq';

$$=\rho\sin\frac{1}{2}\beta\,.\,\frac{\cos^2\frac{1}{2}\theta}{\sin\frac{1}{2}\theta}\,d\theta.k. \quad \text{. . . . . (1)}$$

16. The degree of error incurred in thus taking k for k' may be judged of by applying the formula to an extreme case. For example, let us suppose that the surface of

the earth at this part is a plane passing through the middle point of the elementary base qQq', and sloping upwards towards the side on which A lies. This is an extreme case; for the actual height will be extremely small near the middle of qq', that is, in parts where the variations of $\cos\psi$ are least; and the actual heights will be greatest towards q and q', that is, in parts where the variations of $\cos\psi$ are greatest. Hence the error incurred in replacing the actual heights by their average will be much greater in this than in a more general case. Now in this extreme instance

$$k'=(c-c\cos\psi)\tan\eta,$$

c being the distance of qQq' from A, and η the inclination of the plane to the horizon.

Hence the true value of the integral

$$\int_{-\frac{1}{2}\beta}^{\frac{1}{2}\beta} k'\cos\psi d\psi=c\tan\eta\int_{-\frac{1}{2}\beta}^{\frac{1}{2}\beta}(\cos\psi-\cos^2\psi)d\psi$$

$$=c\tan\eta\left(2\sin\frac{1}{2}\beta-\frac{1}{2}\beta-\frac{1}{2}\sin\beta\right)$$

$$=c\tan\eta\left(\frac{1}{24}\beta^3-\frac{7}{1920}\beta^5+\ldots\right).$$

And the approximate value of the integral

$$=2\sin\frac{1}{2}\beta.k=2\sin\frac{1}{2}\beta.\int_{-\frac{1}{2}\beta}^{\frac{1}{2}\beta}(c-c\cos\psi)\tan\eta d\psi\div\beta$$

$$=\frac{2c}{\beta}\sin\frac{1}{2}\beta\tan\eta\left(\beta-2\sin\frac{1}{2}\beta\right)$$

$$=c\tan\eta\left(\frac{1}{24}\beta^3-\frac{13}{5760}\beta^5+\ldots.\right).$$

Hence the ratio of the true value to the approximate value $=1-\frac{1}{120}\beta^2$. If $\beta=30°$, the width I intend to give to the lunes, this ratio differs from unity by the insignificant fraction 0·00228, which we may well neglect; and if this is the smallness of the error in such an extreme case, we may consider that the approximation will in the general case differ from the true value by an inappreciable quantity.

17. To find the attraction of the whole mass comprised within the compartment, the expression (1) in art. 15. should be integrated with respect to θ from $\theta=\alpha$ to $\theta=\alpha+\varphi$. But to do this we ought to know what function k is of θ. There is, however, no known law of connexion between them. We must therefore resort to some other means. According to the law of dissection, which I shall in the end adopt, the value of $\frac{\cos^2\frac{1}{2}\theta}{\sin\frac{1}{2}\theta}$ does not vary much within the limits of a single compartment, that is, between the limits $\theta=\alpha$ and $\theta=\alpha+\varphi$. Indeed its middle value, when $\theta=\alpha+\frac{1}{2}\varphi$, is only one-fifteenth part smaller than its greatest value, when $\theta=\alpha$, and

only one-sixteenth part greater than its smallest value, when $\theta=\alpha+\phi$. I shall therefore take this middle value instead of the variable value, and the expression (1.) becomes integrable. Suppose h is the mean value of k from $\theta=\alpha$ to $\theta=\alpha+\phi$; then

Attraction of whole mass standing on the compartment BB′C′C

$$=\varrho \sin\frac{1}{2}\beta.\frac{\phi\cos^2\left(\frac{1}{2}\alpha+\frac{1}{4}\phi\right)}{\sin\left(\frac{1}{2}\alpha+\frac{1}{4}\phi\right)}h. \quad \ldots \ldots \ldots \quad (2.)$$

By this approximation the attraction of the nearer and narrower part of the mass is made a little too small, and that of the further and wider part a little too large. These errors tend to counterbalance each other; and the residual error, if any, will be very trifling. In some of the compartments the compensation may be exact; in others, in excess; in others, in defect—according to the variations of k. So that taking all the masses on the lune, the probabilities are that the compensation on the whole will be perfect, and that no error will be incurred.

18. That the extreme values of $\dfrac{\cos^2\frac{1}{2}\theta}{\sin\frac{1}{2}\theta}$ do not differ more than one-fifteenth and one-sixteenth from its middle value appears as follows:—

According to the law of dissection, which I shall soon adopt,

$\phi=\frac{1}{10}\alpha$, when α is very small;

$\phi=\frac{1}{9}\alpha$, when α is about 38°;

$\phi=\frac{1}{8}\alpha$, when α is about 52° 30′;

$\phi=\frac{1}{7}\alpha$, when α is about 65°;

$\phi=\frac{1}{6}\alpha$, when α is about 76°;

$\phi=\frac{1}{5}\alpha$, when α is about 84°;

$\phi=\frac{1}{4}\alpha$, when α is about 130°;

and ϕ has intermediate values for intermediate values of α, and of course the smaller the ratio which ϕ bears to α, the less will be the error in our approximation. Now out of 49 cases in which we have to use the formula (2.),

α is less than 38°, and ϕ is less than $\frac{1}{9}\alpha$,	in	40 cases.
α is less than 52° 30′, and ϕ is less than $\frac{1}{8}\alpha$,	in	3 cases.
α is less than 65°, and ϕ is less than $\frac{1}{7}\alpha$,	in	2 cases.
α is less than 76°, and ϕ is less than $\frac{1}{6}\alpha$,	in	1 case.
α is less than 84°, and ϕ is less than $\frac{1}{5}\alpha$,	in	1 case.
α is less than 130°, and ϕ is less than $\frac{1}{4}\alpha$,	in	1 case.
α is greater than 130°, and ϕ is greater than $\frac{1}{4}\alpha$,	in	1 case.
		49

Let us take, then, the extreme case of the first 40, viz. $\alpha=38°$ and $\phi=\frac{1}{9}\alpha$. Hence $\alpha+\frac{1}{2}\phi=40°\ 6'$, and $\alpha+\phi=42°\ 13'$; and

		Difference.	Ratio to middle value.
Greatest value of $\dfrac{\cos^2\frac{1}{2}\theta}{\sin\frac{1}{2}\theta}$	$=2{\cdot}746$	0·172	$\frac{1}{15}$th.
Middle	$=2{\cdot}574$	0·156	$\frac{1}{16}$th.
Least	$=2{\cdot}418$		

Hence in the most unfavourable of the first 40 cases, viz. that in which ϕ is exactly $=\frac{1}{9}\alpha$, the extreme values will depart only $\frac{1}{15}$th and $\frac{1}{16}$th part from the middle value: and the successive pairs of values on the two sides of the middle will differ less and less as they approach the middle. Suppose that there are n such pairs, and that they form two series in arithmetic progression,

$$-\frac{1}{15}\ldots\ldots 0 \text{ and } 0\ldots\ldots+\frac{1}{16}.$$

The sums of these two series are $-\frac{1}{15}\cdot\frac{n+1}{2}$ and $+\frac{1}{16}\cdot\frac{n+1}{2}$,

and hence the average error or the mean of these

$$=-\left(\frac{1}{15}-\frac{1}{16}\right)\frac{n+1}{2}\div(2n+1)=-\frac{1}{960}\,\frac{2n+2}{2n+1}=-\frac{1}{960},$$

when n is indefinitely increased.

Hence the probable error in this extreme 40th case is in defect, and is about $\frac{1}{1000}$th part of the whole. The probable error in the 9 cases beyond the 40th will be greater than this; but for the 39 cases before the 40th far less and less.

Hence in the whole lune we may fairly consider that no appreciable error will be incurred.

19. I now proceed to select the Law of Dissection, that is, the relation of the lengths of the respective compartments to their distances from A. Upon this depends the simplicity and the success of the method of calculation now proposed.

Let the relation between α and φ be always such, that

$$\frac{\varphi\cos^2\left(\frac{1}{2}\alpha+\frac{1}{4}\varphi\right)}{\sin\left(\frac{1}{2}\alpha+\frac{1}{4}\varphi\right)}=\text{a numerical constant}=c.$$

To fix the value of this constant I shall make $\varphi=\frac{1}{10}\alpha$ when φ and α are indefinitely small. Hence, expanding in powers of φ and α,

$$c=\frac{\varphi}{\frac{1}{2}\alpha+\frac{1}{4}\varphi}=\frac{1}{5+\frac{1}{4}}=\frac{4}{21},$$

and the Law of Dissection of the earth's crust is expressed by the equation

$$\frac{\varphi\cos^2\left(\frac{1}{2}\alpha+\frac{1}{4}\varphi\right)}{\sin\left(\frac{1}{2}\alpha+\frac{1}{4}\varphi\right)}=\frac{4}{21}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3.)$$

This reduces the formula (2.) to the following:—Attraction of mass standing on any compartment

$$=\frac{4}{21}\varrho\sin\frac{1}{2}\beta.h, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4.)$$

which depends simply upon the average height (h) of the surface of the mass above the surface through A, and not at all upon the distance of the compartment from A.

20. It is in this that the remarkable simplicity of the method consists. We have but to calculate the angles from the Law of Dissection (3.), and lay down the circles and the lines diverging from A upon a good map on which the elevations and depressions are marked, and the attractions of the several masses standing on the compartments thus marked out will be given by the formula (4.) at once, when we determine upon their average elevations.

21. Let D be the mean density of the earth, which has been finally fixed at 5·66 of distilled water by the recent experiments of the late Mr. BAILY; r the radius of the earth = 4000 miles; g the measure of gravity; then

$$g=\frac{4.\pi}{3}\text{D}.r$$

$$\text{D}=\frac{3}{4\pi}\ \frac{g}{r}.$$

Hence formula (4.) becomes,—Attraction of mass standing on any one compartment

$$=\frac{4}{21}\frac{\rho}{D}D\sin\frac{1}{2}\beta.h$$

$$=\frac{1}{7\pi}\frac{\rho}{D}\sin\frac{1}{2}\beta.\frac{h}{r}.g.$$

Let h be expressed in parts of a mile; ϱ being the density of the superficial crust of the earth, we shall take $=2\cdot75$, which is the density assigned to the mountain Schehallien; reducing to numbers, we have

Attraction of mass standing on any one compartment

$$=0\cdot000005523\times h\sin\frac{1}{2}\beta.g. \quad \ldots\ldots \quad (5.)$$

This gives the attraction of each mass in terms of gravity.

22. We may from this easily deduce the deflection of the plumb-line caused by the attraction of the mass; for the tangent of deflection evidently equals the expression (5.) divided by g, by the simple law of the resolution of forces. Hence

$$\text{Tangent of deflection }=0\cdot000005523\times h\sin\frac{1}{2}\beta$$

$$=\tan(1''\cdot1392)\times h\sin\frac{1}{2}\beta;$$

∴ Deflection of the plumb-line caused by the mass standing on any one compartment

$$=1''\cdot1392\,h\sin\frac{1}{2}\beta. \quad \ldots\ldots \quad (6.)$$

23. It remains to calculate the dimensions of the successive compartments as indicated by the Law of Dissection which I have adopted. The equation which expresses the law cannot be solved directly; we must therefore resort to approximation or trial. All pairs of values we thus find for α and ϕ must satisfy the equation expressing the law. That equation becomes, on our replacing the arc ϕ by the angle ϕ,

$$\phi^0=\frac{4}{21}\frac{180}{\pi}\frac{\sin\left(\frac{1}{2}\alpha+\frac{1}{4}\phi\right)}{\cos^2\left(\frac{1}{2}\alpha+\frac{1}{4}\phi\right)},$$

or

$$\phi=\log^{-1}\left\{\begin{array}{l}11\cdot0379639\\ +\log\sin\left(\frac{1}{2}\alpha+\frac{1}{4}\phi\right)\\ -2\log\cos\left(\frac{1}{2}\alpha+\frac{1}{4}\phi\right)\end{array}\right\} \quad \ldots\ldots \quad (7.)$$

This is the test which all corresponding values of α and ϕ must satisfy.

24. The solution of equation (3.) expressing the law may be facilitated, for values of α not exceeding 38°, by expansion and approximation. Expand in powers of α and ϕ, and it becomes

$$\phi=\frac{4}{21}\left(\frac{\alpha}{2}+\frac{\phi}{4}\right)\left\{1-\frac{1}{6}\left(\frac{\alpha}{2}+\frac{\phi}{4}\right)^2+\left(\frac{\alpha}{2}+\frac{\phi}{4}\right)^2\right\}$$

$$=\frac{2}{21}\left(\alpha+\frac{\phi}{2}\right)\left\{1+\frac{5}{6}\left(\frac{\alpha}{2}+\frac{\phi}{4}\right)^2\right\};$$

$$\therefore \frac{\alpha}{\phi}+\frac{1}{2}=\frac{21}{2}\left\{1-\frac{5}{6}\left(\frac{\alpha}{2}+\frac{\phi}{4}\right)^2\right\}$$

$$\frac{\alpha}{\phi}=10\left\{1-\frac{7}{8}\left(\frac{\alpha}{2}+\frac{\phi}{4}\right)^2\right\}$$

$$\frac{\phi}{\alpha}=\frac{1}{10}\left\{1+\frac{7}{8}\left(\frac{\alpha}{2}+\frac{\phi}{4}\right)^2\right\}$$

$$=\frac{1}{10}\left\{1+\frac{7}{8}\left(\frac{\alpha}{2}+\frac{\alpha}{40}\right)^2\right\}$$

$$=\frac{1}{10}(1+0\cdot2411\,\alpha^2);$$

or, if α be expressed in degrees, $=\frac{1}{10}(1+0\cdot000073\,\alpha^2)$ . . . . (8.)

This formula, as will appear in the issue, may be used for all values of α up to 38° without sensible error.

25. Let $\alpha_1, \alpha_2, \alpha_3 \ldots \phi_1, \phi_2, \phi_3 \ldots$ be the successive values of α and ϕ for the several compartments of a lune. Then these are connected by the equations

$$\alpha_2=\alpha_1+\phi_1, \quad \alpha_3=\alpha_2+\phi_2, \ldots\ldots$$

As the hills do not begin to rise north and north-east of the station Kaliana till between a distance of three-quarters and a whole degree, I shall assume* the first value of α (viz. α_1)$=0^\circ{\cdot}75$.

* If we wish to apply this method of calculation to any other particular case, we may make any other assumption we please regarding the first value of α, so long as we take care not to apply the method to calculate the attraction on a station *too near* to elevated ground. In short, we must see that the angle ω in art. 14. is sufficiently small to be neglected in the formula therein deduced. If it be not, we must calculate the attraction of the nearer masses by a direct method.

We may use the formula (8.) we have just deduced to calculate the values of α and ϕ nearer to A than α_1. In this way we shall obtain the following series of values, writing them backward from α_1 towards A.

$\alpha_0 = 0^\circ{\cdot}6818$	$\phi_0 = 0^\circ{\cdot}06818$
$\alpha_{-1}=0\cdot6198$	$\phi_{-1}=0\cdot06198$
$\alpha_{-2}=0\cdot5635$	$\phi_{-2}=0\cdot05635$
$\alpha_{-3}=0\cdot5122$	$\phi_{-3}=0\cdot05122$
$\alpha_{-4}=0\cdot4657$	$\phi_{-4}=0\cdot04657$
$\alpha_{-5}=0\cdot4234$	$\phi_{-5}=0\cdot04234$
$\alpha_{-6}=0\cdot3849$	$\phi_{-6}=0\cdot03849$
$\alpha_{-7}=0\cdot3599$	$\phi_{-7}=0\cdot03599$
$\alpha_{-8}=0\cdot3181$	$\phi_{-8}=0\cdot03181$
$\alpha_{-9}=0\cdot2892$	$\phi_{-9}=0\cdot02892$
$\alpha_{-10}=0\cdot2629$	$\phi_{-10}=0\cdot02629$
$\alpha_{-11}=0\cdot2390$	$\phi_{-11}=0\cdot02390$
$\alpha_{-12}=0\cdot2172$	$\phi_{-12}=0\cdot02172$
$\alpha_{-13}=0\cdot1975$	$\phi_{-13}=0\cdot01975$
$\alpha_{-14}=0\cdot1795$	$\phi_{-14}=0\cdot01795$
$\alpha_{-15}=0\cdot1632$	$\phi_{-15}=0\cdot01632$
$\alpha_{-16}=0\cdot1484$	$\phi_{-16}=0\cdot01484$
$\alpha_{-17}=0\cdot1349$	$\phi_{-17}=0\cdot01349$
&c.	&c.

The formula (8.) then gives the following successive values :—

	Distance from A of the nearer ends of the compartments.	Length of the compartments.	Distance from A of the middle points of the compartments.
1.	$\alpha_1 = 0^\circ{\cdot}75$	$\phi_1 = 0^\circ{\cdot}075$	$0^\circ{\cdot}787$
2.	$\alpha_2 = 0{\cdot}825$	$\phi_2 = 0{\cdot}0825$	0·866
3.	$\alpha_3 = 0{\cdot}907$	$\phi_3 = 0{\cdot}0907$	0·949
4.	$\alpha_4 = 0{\cdot}998$	$\phi_4 = 0{\cdot}0998$	1·048
5.	$\alpha_5 = 1{\cdot}098$	$\phi_5 = 0{\cdot}1098$	1·153
6.	$\alpha_6 = 1{\cdot}208$	$\phi_6 = 0{\cdot}1208$	1·268
7.	$\alpha_7 = 1{\cdot}329$	$\phi_7 = 0{\cdot}1329$	1·395
8.	$\alpha_8 = 1{\cdot}461$	$\phi_8 = 0{\cdot}1461$	1·534
9.	$\alpha_9 = 1{\cdot}607$	$\phi_9 = 0{\cdot}1607$	1·687
10.	$\alpha_{10} = 1{\cdot}768$	$\phi_{10} = 0{\cdot}1768$	1·856
11.	$\alpha_{11} = 1{\cdot}945$	$\phi_{11} = 0{\cdot}1945$	2·042
12.	$\alpha_{12} = 2{\cdot}140$	$\phi_{12} = 0{\cdot}2140$	2·247
13.	$\alpha_{13} = 2{\cdot}353$	$\phi_{13} = 0{\cdot}2353$	2·472
14.	$\alpha_{14} = 2{\cdot}589$	$\phi_{14} = 0{\cdot}2589$	2·719
15.	$\alpha_{15} = 2{\cdot}848$	$\phi_{15} = 0{\cdot}2848$	2·990
16.	$\alpha_{16} = 3{\cdot}133$	$\phi_{16} = 0{\cdot}3133$	3·289
17.	$\alpha_{17} = 3{\cdot}446$	$\phi_{17} = 0{\cdot}3446$	3·616
18.	$\alpha_{18} = 3{\cdot}790$	$\phi_{18} = 0{\cdot}3790$	3·980
19.	$\alpha_{19} = 4{\cdot}170$	$\phi_{19} = 0{\cdot}4170$	4·378
20.	$\alpha_{20} = 4{\cdot}586$	$\phi_{20} = 0{\cdot}4586$	4·813
21.	$\alpha_{21} = 5{\cdot}046$	$\phi_{21} = 0{\cdot}5046$	5·298
22.	$\alpha_{22} = 5{\cdot}550$	$\phi_{22} = 0{\cdot}5550$	5·828
23.	$\alpha_{23} = 6{\cdot}106$	$\phi_{23} = 0{\cdot}6106$	6·408
24.	$\alpha_{24} = 6{\cdot}716$	$\phi_{24} = 0{\cdot}6716$	7·054
25.	$\alpha_{25} = 7{\cdot}388$	$\phi_{25} = 0{\cdot}7388$	7·707
26.	$\alpha_{26} = 8{\cdot}127$	$\phi_{26} = 0{\cdot}8127$	8·533
27.	$\alpha_{27} = 8{\cdot}939$	$\phi_{27} = 0{\cdot}8939$	9·386
28.	$\alpha_{28} = 9{\cdot}833$	$\phi_{28} = 0{\cdot}9833$	10·324
29.	$\alpha_{29} = 10{\cdot}816$	$\phi_{29} = 1{\cdot}089$	11·360
30.	$\alpha_{30} = 11{\cdot}905$	$\phi_{30} = 1{\cdot}202$	12·506
31.	$\alpha_{31} = 13{\cdot}107$	$\phi_{31} = 1{\cdot}326$	13·770
32.	$\alpha_{32} = 14{\cdot}433$	$\phi_{32} = 1{\cdot}462$	15·211
33.	$\alpha_{33} = 15{\cdot}99$	$\phi_{33} = 1{\cdot}620$	16·80
34.	$\alpha_{34} = 17{\cdot}61$	$\phi_{34} = 1{\cdot}800$	18·51
35.	$\alpha_{35} = 19{\cdot}41$	$\phi_{35} = 1{\cdot}992$	20·40
36.	$\alpha_{36} = 21{\cdot}40$	$\phi_{36} = 2{\cdot}211$	22·50
37.	$\alpha_{37} = 23{\cdot}61$	$\phi_{37} = 2{\cdot}456$	24·83
38.	$\alpha_{38} = 26{\cdot}06$	$\phi_{38} = 2{\cdot}734$	27·43
39.	$\alpha_{39} = 28{\cdot}79$	$\phi_{39} = 3{\cdot}054$	30·31
40.	$\alpha_{40} = 31{\cdot}84$	$\phi_{40} = 3{\cdot}419$	33·55
41.	$\alpha_{41} = 35{\cdot}26$	$\phi_{41} = 3{\cdot}600$	37·06
42.	$\alpha_{42} = 38{\cdot}86$	$\phi_{42} = 4{\cdot}314$	41·01

When part of the attracting mass is near, and therefore ω is not so small that it may be neglected (as in art. 14.)—which will be the case in carrying the survey into the mountainous regions—a different formula must be used for calculating the deflection of the plumb-line caused by the nearer parts of the attracting mass.

Let, as before, the mass be divided by lunes, which for the parts now under consideration must be narrow; let ω and ω' be the angles which the highest and lowest points of a small vertical prism, reaching from any point of the surface down to the sea-level, makes with the horizontal line at the eye of the observer. Let θ be the horizontal distance from the observer of the middle line of the prism, in degrees; β the width of the lune, in degrees; l the length of the compartment on which the prism stands, in miles; r the radius of the earth, in miles. Then the Lemma in art. 13. leads to the following exact formula :—

26. That the formula (8.) which I have been using to determine these values does not, thus far, lead to material error may be shown by substituting the 42nd pair of values in the test given by formula (7.). In this case

$$\alpha=38^{\circ}\cdot86,\quad \varphi=4^{\circ}\cdot314,\quad \frac{1}{2}\alpha+\frac{1}{4}\varphi=20^{\circ}\cdot50=20^{\circ}\ 30',$$

and formula (7.) becomes

$$\varphi=\log^{-1}\left\{\begin{array}{r} 11\cdot0379639 \\ \underline{9\cdot5443253} \\ 20\cdot5822892 \\ \underline{19\cdot9431752} \\ 0\cdot6391140 \end{array}\right\}=4^{\circ}\cdot356.$$

Hence the error$=4^{\circ}\cdot356-4^{\circ}\cdot314=0^{\circ}\cdot042$, and this equals $\frac{1}{103}$rd of the whole.

If the same test be applied to the 40th values of α and φ, the error is only $\frac{1}{305}$th of the whole; and as we pass further back it becomes absolutely evanescent.

27. The remaining values of α and φ, after the 42nd as above determined, we must find by solving equation (3.) by trial and testing the values by formula (7.).

Horizontal attraction of the prism on the station of the observer

$$=\frac{3}{4}\frac{\rho}{\mathrm{D}}\frac{\beta}{180}\frac{l}{r}\left\{\left(1+\tan\frac{1}{2}\theta\tan\frac{1}{2}\omega\right)\sin\omega+\left(1+\tan\frac{1}{2}\theta\tan\frac{1}{2}\omega'\right)\sin\omega'\right\}\cos\frac{1}{2}\theta.$$

In this $\tan\frac{1}{2}\theta\tan\frac{1}{2}\omega$ and $\tan\frac{1}{2}\theta\tan\frac{1}{2}\omega'$ may be neglected as quite insensible; for $\tan\frac{1}{2}\omega$ and $\tan\frac{1}{2}\omega'$ can neither of them be greater than 1, and in that case $\theta=0$; and when they have any other sensible value, $\tan\frac{1}{2}\theta$ is of insensible magnitude. So also as θ is never made larger than 1°, or so large, in the application of this formula, $\cos\frac{1}{2}\theta$ may be put $=1$; and the formula becomes

Horizontal attraction of the prism on the station of the observer

$$=\frac{3}{4}\frac{\rho}{\mathrm{D}}\frac{\beta}{180}\frac{l}{r}(\sin\omega+\sin\omega').$$

The values of l need not follow any law, but may be chosen in each lune according to the form of the vertical section; some values being long and some short, according as the variations of ω and ω' are slow or rapid. The angles ω and ω' must be found as follows:—The lune having been divided into compartments, the average height and depth of the top and bottom of the prism standing on each compartment, above and below the observer's horizon, must be divided by the horizontal distance of the prism from the observer. This will give the tangents of the angles ω and ω', whence the sines may be found.

The above expression must be thus calculated for all the compartments: the whole added together gives the attraction of the mass standing on the portion of the lune to which this method is to be applied. This sum, multiplied by the cosine and sine of the azimuth, will give the attraction in the meridian and in the prime vertical. The same being done all round the circle, the resultant attraction and the azimuth of the plane are easily found; whence the deflection is known, and the various angles of observation may be corrected.

In using the method of the text for the parts beyond, the heights $h_1 h_2 \ldots\ldots$ must be measured from the sea-level, and not from the surface, parallel to the sea-level, passing through the station of the observer. This *is* done in the text in the case of Kaliana, because it appears that below the level of that place there are no variations of surface sufficient to produce any sensible alteration in the attraction of the whole mass. This, however, will not be the case with stations in the mountains.

The following applications of the test show that the values I shall now write down are correct:—

$$\alpha_{43}=43^{\circ}\cdot17,\ \varphi_{43}=4^{\circ}\cdot98;\ \varphi=\log^{-1}\left\{\begin{array}{r}11\cdot0379639\\ \underline{9\cdot5888296}\\ 20\cdot6267935\\ \underline{19\cdot9291416}\\ 0\cdot6976519\end{array}\right\}=4^{\circ}\cdot985.$$

$$\alpha_{44}=48^{\circ}\cdot15,\ \varphi_{44}=5^{\circ}\cdot783;\ \varphi=\log^{-1}\left\{\begin{array}{r}11\cdot0379639\\ \underline{9\cdot6351413}\\ 20\cdot6731052\\ \underline{19\cdot9108962}\\ 0\cdot7622090\end{array}\right\}=5^{\circ}\cdot784.$$

$$\alpha_{45}=53^{\circ}\cdot93,\ \varphi_{45}=6^{\circ}\cdot80;\ \varphi=\log^{-1}\left\{\begin{array}{r}11\cdot0379639\\ \underline{9\cdot6808891}\\ 20\cdot7188530\\ \underline{19\cdot8864756}\\ 0\cdot8323774\end{array}\right\}=6^{\circ}\cdot80.$$

$$\alpha_{46}=60^{\circ}\cdot73,\ \varphi_{46}=8^{\circ}\cdot21;\ \varphi=\log^{-1}\left\{\begin{array}{r}11\cdot0379639\\ \underline{9\cdot7292234}\\ 20\cdot7671873\\ \underline{19\cdot8528620}\\ 0\cdot9143253\end{array}\right\}=8^{\circ}\cdot21.$$

If h be the height above the observer's eye of the top of the prism, and h' the depth of the bottom of it below, then (see figure in art. 13)

$$\frac{h+r}{r}=\frac{\mathrm{OR}}{\mathrm{OA}}=\frac{\sin \mathrm{OAR}}{\sin \mathrm{ORA}}=\frac{\cos\left(\frac{1}{2}\theta-\omega\right)}{\cos\left(\frac{1}{2}\theta+\omega\right)};$$

$$\therefore \frac{h}{r}=\frac{2\sin\omega\sin\frac{1}{2}\theta}{\cos\left(\frac{1}{2}\theta+\omega\right)}=2\sin\omega\tan\frac{1}{2}\theta;$$

since ω is extremely small in the parts to which the method of the text is to be applied.

So also
$$\frac{h'}{r}=2\sin\omega'\tan\frac{1}{2}\theta.$$

Substituting for $\sin\omega$ and $\sin\omega'$ from these in the exact formula above (which applies to all cases), and neglecting excessively small quantities,

Horizontal attraction of the prism on the station of the observer

$$=\frac{3}{4}\frac{\rho}{\mathrm{D}}\frac{\beta}{180}\frac{l}{r}\cdot\cos\frac{1}{2}\theta\cot\frac{1}{2}\theta\cdot\frac{h+h'}{2r};$$

or the attraction of the parts to which the method of the text applies is found by taking the sum of the heights $(h+h')$, or the whole height from the sea-level up to the surface of the attracting mass. This, it will be observed, is the same whatever be the height of the station of observation. In fact, the horizontal attraction of the mass, situated so far off as 50 miles and more, upon any station, even though in the mountains, cannot differ in any appreciable degree from that upon the point where the vertical line at the station cuts the sea-level below.

$$\alpha_{47}=68^{\circ}\cdot 94,\ \phi_{47}=10^{\circ}\cdot 33;\ \phi=\log^{-1}\left\{\begin{array}{r} 11\cdot 0379639 \\ \underline{9\cdot 7799655} \\ 20\cdot 8179294 \\ \underline{19\cdot 8041256} \\ 1\cdot 0138038 \end{array}\right\}=10^{\circ}\cdot 33.$$

$$\alpha_{48}=79^{\circ}\cdot 27,\ \phi_{48}=14^{\circ}\cdot 03;\ \phi=\log^{-1}\left\{\begin{array}{r} 11\cdot 0379639 \\ \underline{9\cdot 8348646} \\ 20\cdot 8728285 \\ \underline{19\cdot 7263656} \\ 1\cdot 1464629 \end{array}\right\}=14^{\circ}\cdot 01.$$

$$\alpha_{49}=93^{\circ}\cdot 30,\ \phi_{49}=23^{\circ}\cdot 38;\ \phi=\log^{-1}\left\{\begin{array}{r} 11\cdot 0379639 \\ \underline{9\cdot 8995636} \\ 20\cdot 9375275 \\ \underline{19\cdot 5685648} \\ 1\cdot 3689627 \end{array}\right\}=23^{\circ}\cdot 38.$$

$\alpha_{50}=116^{\circ}\cdot 68$, and ϕ_{50} reaches beyond the antipodes. For let us find α, supposing that ϕ just reaches to the antipodes; then $\alpha+\phi=180^{\circ}$, $\frac{1}{2}\alpha+\frac{1}{4}\phi=90^{\circ}-\frac{1}{4}\phi$, and equation (3.) gives

$$\phi=\frac{4}{21}\,\frac{\cos\frac{1}{4}\phi}{\sin^{2}\frac{1}{4}\phi},$$

an equation which is satisfied by $\phi=82^{\circ}\ 30'$, and therefore the corresponding value of α would be $180^{\circ}-82^{\circ}\ 30'=97^{\circ}\ 30'$. But this is *less than* α_{50} as above determined. Hence ϕ_{50} will reach, as I have said, beyond the antipodes. The reason why the compartments increase in length with such excessive rapidity when they are more than 90° from A, is, not only the increasing distance of the attracting mass, but their convergency towards the antipodes and the consequent contraction of their width, and also the great angle at which the attraction acts with the tangent at A, and the consequent smallness of its resolved part along that line.

28. The following list of values, then, of α and ϕ will form the continuation of those in Art. 25 :—

	Distance from A of the nearer ends of the compartments.	Lengths of the compartments.	Distance from A of the middle points of the compartments.
43.	$\alpha_{43}=43^{\circ}\cdot 17$	$\phi_{43}=4^{\circ}\cdot 980$	$45^{\circ}\cdot 66$
44.	$\alpha_{44}=48\cdot 15$	$\phi_{44}=5\cdot 783$	51·04
45.	$\alpha_{45}=53\cdot 93$	$\phi_{45}=6\cdot 800$	57·33
46.	$\alpha_{46}=60\cdot 73$	$\phi_{46}=8\cdot 210$	64·83
47.	$\alpha_{47}=68\cdot 94$	$\phi_{47}=10\cdot 330$	74·10
48.	$\alpha_{48}=79\cdot 27$	$\phi_{48}=14\cdot 030$	86·28
49.	$\alpha_{49}=93\cdot 30$	$\phi_{49}=23\cdot 380$	104·99
50.	$\alpha_{50}=116\cdot 68$	ϕ_{50} is imperfect.	

29. These distances should be laid down and the circles drawn on a map or globe; and nothing remains to be done, but to ascertain the average heights of the masses standing on the compartments thus drawn.

If the surface of any of these masses is very irregular, several vertical sections should be taken in directions most favourable for giving a fair average. One convenient method of using such sections is, after laying them down on a scale on good paper, to cut them out, weigh them, and compare the weight with that of a portion of the same kind of paper of known dimensions on the same scale. The resulting number divided by the aggregate length of all the sections will give the average height. It may be convenient to use different scales for the vertical and horizontal measures: this may be done if it be carefully attended to in carrying out this method.

30. It will be evident that mountain ranges will assume a less importance in this calculation as they are more distant from A, since they will stand on a larger compartment; and therefore when in imagination levelled down to cover the whole compartment and give the average height, they will stand at a much less altitude. It is for this reason that a knowledge of extensive table-lands of considerable elevation, and of the elevated channels of rivers, is of far more importance in this calculation—especially in the remoter parts—than of mountain peaks and mountain ridges.

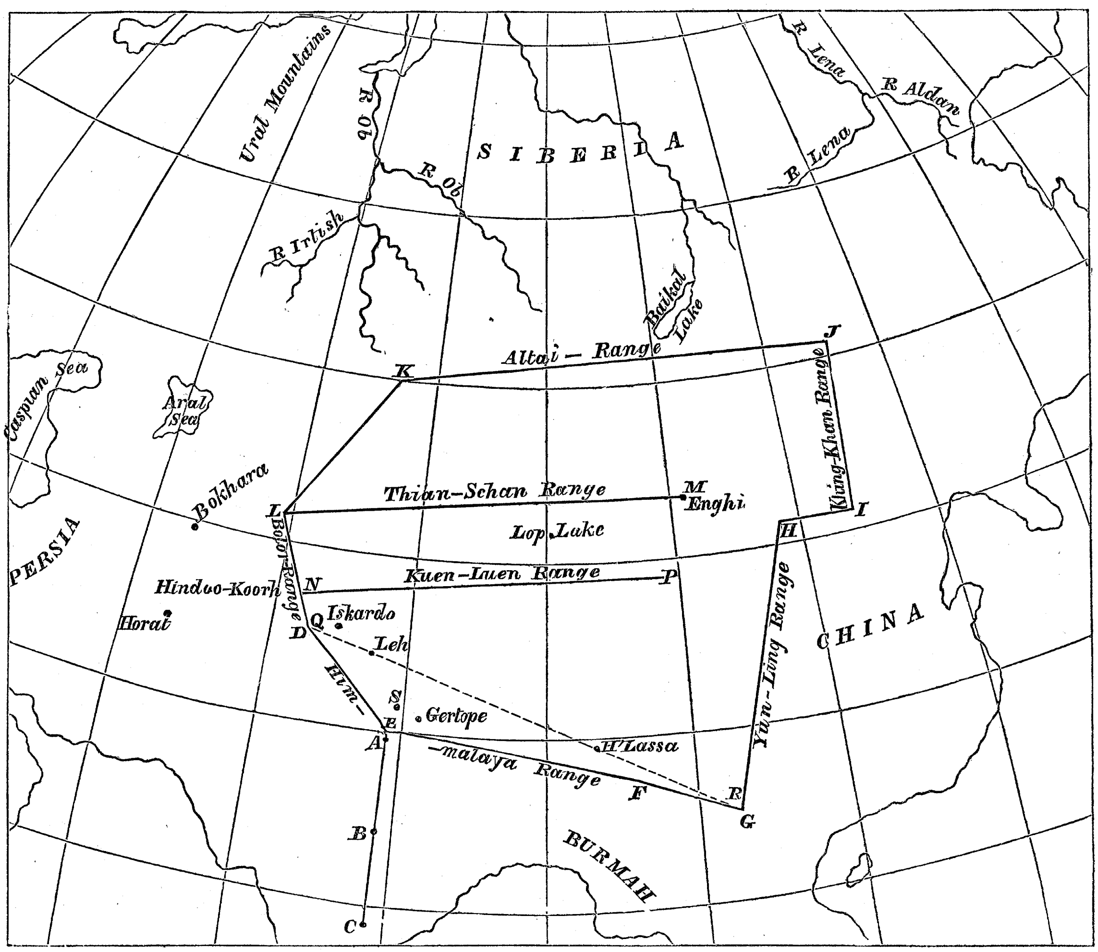
II. *Approximation to the amount of mountain attraction at the two extremities and the middle station of the Great Indian Arc of the Meridian between latitudes* 18° 3′ 15″ *and* 29° 30′ 48″.

31. The complete application of the method I have developed requires a full survey of the earth's superficies. In the absence, however, of sufficiently accurate and extensive information to make an exact calculation, I propose now to use such data as I have been able to gather—chiefly from books on geography and HUMBOLDT's works, as well as the published Maps of the Indian Survey—to obtain an approximation to the amount of attraction on the plumb-line on the Indian arc. I regret that absence from India—which is the occasion of my finding leisure to draw up this paper—prevents my consulting HUMBOLDT's Map of Central Asia (published in 1842), although extracts from his 'Aspects of Nature' will in part supply the want.

32. Fig. 4 represents an outline of the continent of Asia. A, B, C are the northern, middle, and southern stations of the Great Arc I am about to consider. These stations are Kaliana in latitude 29° 30′ 48″, Kalianpur in 24° 7′ 11″, and Damargida in 18° 3′ 15″. The longitude of the arc is about 77° 42′.

The polygonal figure DEFGHIJKLD, which for the convenience of a name I shall call the *Enclosed Space*, marks out the boundary of an irregular mass, which is the only part of the earth's surface that appears to have a sensible effect on the plumb-line in India. DEF is the Himalaya range, having a bend at E from north-west on the left to east by south on the right. FG is a range running to the table-land of Yu-nan in lat. 25° and long. 103°. GH is the range of the Yun-Ling mountains, in which there are many peaks of perpetual snow. HI is the Inshan range. IJ is the Khing-Khan range, very steep on the east side, not so on the west: the passes are said to be 5525 feet above the sea. JK is the Altai range, the highest peak of which is 10,800 feet, the average height is 6000: the range declines towards the east. KL was once thought to be a range of mountains, but it is now found to be

Fig. 4.



a line of broken country. LD is the Bolor range, rising to an elevation similar to that of the Hindoo Koosh. There are, besides these, two ranges of high mountains running east into the Enclosed Space, parallel to the Altai and Southern Himalayas, viz. LM, the Thian-Schan range, or Celestial Mountains, and NP the Kuen-Luen range, being a continuation of the Hindoo Koosh, which rises from an altitude of 2558 feet near Herat to about 20,000 near N, where it meets the Bolor range. It is with the elevation of the Enclosed Space itself that we are principally concerned, since, as observed in art. 30, ranges of mountains have not so important an influence, when distant, as table-lands of elevation.

33. Before describing the country within these limits, I will give a general sketch of the parts which lie outside, from which it will appear that we may confine our calculations to this Enclosed Space. As the elevation of the Station A. is about 1000 feet above the level of the sea, we shall take at this height the surface of reference parallel to the sea-level from which all altitudes are to be measured in our calculations. The stations Kalianpur and Damargida are higher than Kaliana. In making the Kaliana-level our basis, while we consider A to be the station Kaliana itself, B and C must be considered to be points vertically below Kalianpur and Damargida and situated on the Kaliana-level. So that our calculation of the attraction at the middle and south end of the arc will strictly speaking apply to these points. The difference, however, of the attraction at these points and the stations under which they lie will be utterly inappreciable, unless the country around B and C, which we have left out of the account as having no sufficient elevation, produce a sensible effect*.

34. HUMBOLDT says in his 'Aspects of Nature,' that to the *lowlands* belong almost the whole of Northern Asia to the north-west of the volcanic range of the Thian-Schan, the steppes to the north of the Altai and of the Sayan range, the countries which extend from the mountains of Bolor or Bulyt-Tagh, from the upper Oxus to the Caspian, and from Tenghir or the Balkhash Lake through the Kirghis steppe, towards the sea of Aral and southern extremity of the Ural Mountains. "As compared," he adds, "with high plains of 6000 and 10,000 feet above the level of the sea, it may well be permitted to use the expression *lowlands* for flats of little more than 200 to 1200 feet of elevation." Again, "the plains through which the upper Irtish flows [rising near K in fig. 4 and running north towards the north sea] are scarcely raised 850, or at most 1170 feet above the level of the sea." The mountains about the river Aldan on the north-east and the Ural Mountains to the north-north-west—the former not averaging more than 2000 feet in height, and the latter 4000—can have no influence owing to their great distance, as well as small elevation. In short, the whole country to the north, north-west, and north-east of the Enclosed Space is of so inconsiderable a height above the sea, that it may be left out of our reckonings. The same may be said of all to the west. Thus Sir ALEXANDER BURNES assigns to Bokhara an elevation of only 1190 feet. The level of the Caspian is 83 feet below the Black Sea, and is the centre of an extensive depression—the Caucasus excepted. The Hindoo Koosh runs off at N, a short distance to the west, but soon descends into the generally-plain country, and at Herat is only 2558 feet high. The only

* The actual heights above the sea are thus given in Colonel EVEREST's work:—

Height of Kaliana above the sea	942·3 feet.
Height of Kalianpur above the sea	1878·2 feet.
Height of Damargida above the sea	2090·5 feet.

great elevations in these parts are the Hindoo Koosh and the Caucasus; but these are of so small an extent and width, that when levelled down they will not sensibly raise the average height of the large compartments in which they stand. In Arabia I believe there are some moderately elevated table-lands. But their effect will be somewhat lessened by the intervening ocean, as its density is only two-fifths that of rock. Moreover the effect will be sensible, if sensible at all, only in the direction of the prime-vertical at the three stations, and not in the meridian, which is the only direction of importance to our calculation. The effect is also in part counterbalanced by those parts of the countries of Burmah, Malacca, Siam, and China, which lie outside the Enclosed Space, and are on the opposite side of the Indian Arc from Arabia. The effect of the Ghat Mountains on the west of India, and the table-land of Central and Southern India, will be counteracted in part by the extensive ocean beyond them. Their effect upon the northern Station A. will be inconsiderable; and with regard to B. and C., what effect they may have will be chiefly in the prime-vertical. So to the east of the Enclosed Space. The parts of China beyond it, in which there is only a mountain range on the sea-coast and of no considerable elevation or extent, will have but a feeble influence. Hence these regions around the Enclosed Space may be left out of the account. And those lying still further off and running to the antipodes may also be passed over, as the distance of their several parts is so great compared with the distances of A, B, and C from each other, that the resultant attraction of those regions, whatever high table-lands may occur in them, must be almost precisely the same at all three of the stations. It would not be difficult indeed to show that this resultant attraction is itself of imperceptible amount.

We may fairly conclude, then, that the disturbing cause lies wholly in the enormous mass included within the Enclosed Space, which I shall now describe.

35. The Himalayas rise abruptly from the plains of India to 4000 feet and more, and cover an extensively broken surface some 100 or 200 miles wide, rising to great heights,—perhaps 200 summits exceeding 18,000 feet, and the greatest reaching more than 28,000. The general base on which these peaks rest rises gradually to 9000 or 10,000 feet, where it abuts on the great plateau north of the range. The character of the country to the south of this plateau is much better known than that to the north. If a circle be drawn around A with a radius $=5^{\circ}{\cdot}046$ (the value of α_{21}), it will pass over the highest part of this plateau, just taking in Leh or Ladak within its compass. The general features of the country within this circle may be learnt from the Survey and other maps of India. This portion, then, of the Enclosed Space I shall call the *Known Region,* and the whole that lies beyond it the *Doubtful Region.* By introducing an arbitrary factor into the calculations I shall be able to separate the effects of these two divisions, and to gather what influence our uncertainty in the Doubtful Region has upon the total result.

36. These two divisions join on the great plateau. On this plateau are H'Lassa, according to HUMBOLDT 9590 feet above the sea; Gertrope, 10,500; and Leh or

Ladak, 9995; but this last is "in a hollow, the surrounding plateau rising to 13,430 feet." Iskardo is 6300 feet; but "south of Iskardo the plateau Deotsuh rises to 11,977 feet." "There are, properly speaking," HUMBOLDT observes, "very few plains [in this part of the Enclosed Space now under consideration]; the most considerable are those between Gertope, Daba, Schang-thung, the native country of the shawl goat, and Shipke (10,450 feet),—those around Ladak, which have an elevation of 13,430 English feet, and [as noticed above] must not be confounded with the depression in which the town is situated,—and lastly, the plateau of the sacred lakes of Manasa and Ravanahrada (probably 15,000 feet)...... From many carefully collected measurements of elevation I think I may say," he adds, "that the plateau of Thibet, between 73° and 85° east longitude, does not reach a mean height of 1800 toises, or 11,510 English feet." In addition to these observations, I may add, that the Survey Map of India, No. 65, shows that the bed of the Sutledge at the point marked S in fig. 4 is 10,792 feet.

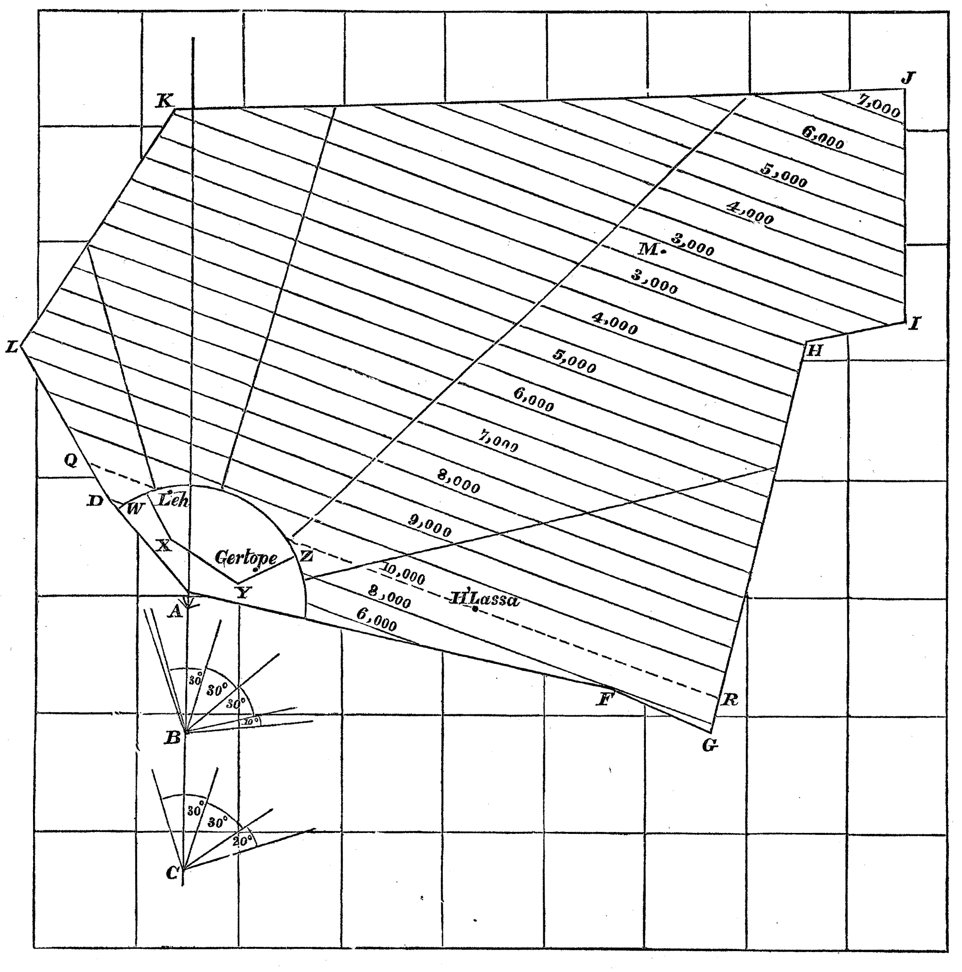
As the result of these data, I shall take the elevation of the dotted line QR, marked in fig. 4, and passing through Leh and H'Lassa, to be 10,000 feet, the greater portion being 11,000 or more, and the extremities somewhat less. This will be rather under the mark than above it.

37. With regard to the parts of the Enclosed Space further removed, HUMBOLDT observes,—"The mean height of this part of Gobi [between the sources of the river Selenga, lat. 50°, long. 102°, and the great wall of China, 600 geographical miles] is barely 4,264 English feet......Enghi is half-way, and is only about 2558 feet." I shall therefore take the point M, which represents the situation of Enghi, to be 2560 feet. I gather from the general account of the country between the great plateau of Thibet and the parts about Enghi and beyond that place, that, although it varies in its surface, it has a general slope down to Enghi, and then a rise again to the mountains. In describing the parts to the north of NP, the Kuen-Luen range, HUMBOLDT states, that between that and the Thian-Schan ranges there is a considerable depression. "CARL ZIMMERMAN," he remarks, "has made it appear extremely probable, that the Tarim depression, that is, the desert between the mountain chains of Thian-Schan and Kuen-Luen, where the steppe-river Tarim-gol empties itself into the Lake of Lop, which used to be described as an Alpine lake, is hardly 1279 English feet above the level of the sea." Again, he informs us that the line KL (in fig. 4), though once supposed to be a chain of mountains, is a line of broken country; and the west of this line we know to be lowland.

38. Guided, then, by such data as I have been able to gather, I assume, as the best general representation of the facts, that the Doubtful Region of the Enclosed Space to the north of QR, and outside the circle about A which bounds the Known Region, slopes gradually from 10,000 feet down to 2560 at M, and then rises again at the same angle to J. And I shall assume that the parts of the same region to the south of QR, and not included in the Known Region, slopes at four times that rate. In

fig. 5 I have laid down the Enclosed Space on a scale, and, for convenience of reference, drawn parallel lines over the whole of the Doubtful Region, marking the elevations of the various parts according to the assumed law.

Fig. 5.



39. The width of the lunes by which the enclosed space is to be traversed I shall make 30°, as this will lead to no material error by making the compartments too wide to obtain a fair average of their height (see art. 16.). In a closer approximation, however, it would be desirable to make the lunes more numerous and narrower. At each of the stations A, B, C, I shall make the middle line of one lune coincide with the meridian. On the right and left limits of the Enclosed Space there will be lunes of a less width than 30°. In short, the lunes will be as follows:—

At A, lune I., width $\beta=24°$, azimuth of middle line $=27°$ west.
At A, lune II., width $\beta=30°$, azimuth of middle line $=0°$.
At A, lune III., width $\beta=30°$, azimuth of middle line $=30°$ east.
At A, lune IV., width $\beta=30°$, azimuth of middle line $=60°$ east.
At A, lune V., width $\beta=20°$, azimuth of middle line $=85°$ east.

These will take in the whole Enclosed Space, both Known and Doubtful Regions. In like manner for the other stations:—

At B, lune I., width $\beta=8°$, azimuth of middle line $=19°$ west.
At B, lune II., width $\beta=30°$, azimuth of middle line $=0°$.
At B, lune III., width $\beta=30°$, azimuth of middle line $=30°$ east.
At B, lune IV., width $\beta=30°$, azimuth of middle line $=60°$ east.
At B, lune V., width $\beta=10°$, azimuth of middle line $=80°$ east.

At C, lune I., width $\beta=30°$, azimuth of middle line $=0°$.
At C, lune II., width $\beta=30°$, azimuth of middle line $=30°$.
At C, lune III., width $\beta=20°$, azimuth of middle line $=55°$ east.

40. By spherical trigonometry and an examination of the maps the following results have been obtained, from which it will be seen which of the quantities $h_0\, h_1\, h_2 \ldots.$ in the several lunes fall within the Enclosed Space (and therefore have any sensible effect), and which in the surrounding low country.

STATION A.

In lune I., from h_5 to h_{20} occur in Known Region, from h_{20} to h_{31} in Doubtful Region.
In lune II., from h_4 to h_{20} occur in Known Region, from h_{21} to h_{35} in Doubtful Region.
In lune III., from h_2 to h_{20} occur in Known Region, from h_{21} to h_{37} in Doubtful Region.
In lune IV., from h_1 to h_{20} occur in Known Region, from h_{21} to h_{40} in Doubtful Region.
In lune V., from h_3 to h_{20} occur in Known Region, from h_{21} to h_{34} in Doubtful Region.

STATION B.

In lune I., only h_{27} occurs in Known Region, from h_{28} to h_{34} in Doubtful Region.
In lune II., from h_{23} to h_{28} occur in Known Region, from h_{29} to h_{37} in Doubtful Region.
In lune III., from h_{22} to h_{26} occur in Known Region, from h_{27} to h_{39} in Doubtful Region.
In lune IV., none occur in Known Region, from h_{27} to h_{39} in Doubtful Region.
In lune V., none occur in Known Region, from h_{32} to h_{37} in Doubtful Region.

STATION C.

In lune I., from h_{30} to h_{33} occur in Known Region, from h_{34} to h_{40} in Doubtful Region.
In lune II., none occur in Known Region, from h_{32} to h_{40} in Doubtful Region.
In lune III., none occur in Known Region, from h_{33} to h_{38} in Doubtful Region.

41. The following is the method I have pursued to determine the heights of the masses standing on the successive compartments which fall within the Known and Doubtful Regions. For the Known Region, I marked out the lunes and their middle lines upon the Survey Maps, and such others as I could procure for regions beyond them, by stretching threads upon them; and for the Doubtful Region I used my own diagram, fig. 5, where maps were not available. I then marked on a slip of paper for each map or diagram, according to the scale of each, the distances from the

attracted station of the middle points of the compartments of each lune, these distances being taken from the third column of numbers in art. 25 and 28. By laying the slip along the middle line of the lune, with one end at the station, with the greatest ease I dotted down on the map the centres of the compartments, and then by an inspection of the heights, recorded on the map, of ranges and especially of the beds of rivers, determined the average height of the mass on each compartment above the level of Station A. In using the diagram fig. 5, I counted the number of centres of compartments which occur, and marked the heights of the first and last lying on the slope, whence by summing an arithmetic series I obtained the sum of the whole heights, without taking the trouble of noting each of them down. The result of these measures is given in the following Tables. H_1 and H_2 represent the sums of the heights in *miles* in the Known and Doubtful Regions. The line E at the bottom of each gives the correction which should be applied to the values of $H\sin\frac{1}{2}\beta\cos Az$, in any given lune, for either the Known or Doubtful Region, if the heights in that lune are all increased or diminished by 100 feet.

TABLE I.—Station A. (Kaliana), *Known Region.*

Heights of the masses in the compartments above A.	Lune I.	Lune II.	Lune III.	Lune IV.	Lune V.
h_1	....	....	....	600 feet	....
h_2	....	....	500 feet	600 feet	....
h_3	....	....	600 feet	1200 feet	300 feet
h_4	....	500 feet	1500 feet	800 feet	300 feet
h_5	300 feet	800 feet	1500 feet	1500 feet	300 feet
h_6	500 feet	1000 feet	1200 feet	1600 feet	300 feet
h_7	800 feet	2000 feet	600 feet	1600 feet	800 feet
h_8	1300 feet	3000 feet	2000 feet	1800 feet	1000 feet
h_9	2300 feet	3000 feet	2500 feet	2500 feet	1500 feet
h_{10}	1800 feet	5000 feet	3000 feet	6000 feet	2000 feet
h_{11}	300 feet	2500 feet	5500 feet	7000 feet	2500 feet
h_{12}	800 feet	4000 feet	7000 feet	7000 feet	3000 feet
h_{13}	800 feet	4000 feet	8500 feet	9000 feet	2500 feet
h_{14}	800 feet	5000 feet	9500 feet	9000 feet	2500 feet
h_{15}	800 feet	5500 feet	9000 feet	9000 feet	2500 feet
h_{16}	800 feet	6000 feet	9500 feet	9000 feet	2500 feet
h_{17}	1300 feet	7000 feet	9500 feet	9500 feet	2500 feet
h_{18}	1800 feet	7500 feet	9500 feet	9500 feet	3000 feet
h_{19}	2800 feet	8300 feet	9500 feet	9500 feet	3000 feet
h_{20}	4800 feet	9500 feet	9500 feet	9500 feet	3000 feet
Sum = ...	22,000 feet	74,600 feet	100,400 feet	106,200 feet	33,500 feet
H_1=	4·167 miles	14·129 miles	19·015 miles	20·114 miles	6·344 miles
$H_1\sin\frac{1}{2}\beta$=	0·866 mile	3·656 miles	4·921 miles	5·202 miles	1·163 mile
$H_1\sin\frac{1}{2}\beta\cos Az.$ =	0·772 mile	3·656 miles	4·262 miles	2·602 miles	0·095 mile
$H_1\sin\frac{1}{2}\beta\sin Az.$ =	−0·393 mile	0·000 mile	2·460 miles	4·223 miles	1·057 mile
E=.....................	0·05614 mile	0·08331 mile	0·08066 mile	0·04901 mile	0·00515 mile

TABLE II.—Station A. (Kaliana), *Doubtful Region.*

Heights of the masses in the compartments above A.	Lune I.	Lune II.	Lune III.	Lune IV.	Lune V.
h_{21}	Three heights, from 7000 feet to 9000 feet	Fifteen heights, from 8700 feet to 2000 feet	Fifteen heights, from 8500 feet to 1500 feet	7000 feet	Nine heights, from 3000 feet to 9000 feet
h_{22}				Seventeen heights, from 9000 feet to 1500 feet	
h_{23}					
h_{24}	Eight heights, from 9000 feet to 7000 feet				
h_{25}					
h_{26}					
h_{27}					
h_{28}					
h_{29}					
h_{30}					Five heights, from 9000 feet to 6000 feet
h_{31}					
h_{32}	....				
h_{33}	....				
h_{34}	....				
h_{35}	....				....
h_{36}	....	....	1500 feet		....
h_{37}	....	....	3500 feet		....
h_{38}	....	....	....		....
h_{39}	....	....	....	1500 feet	....
h_{40}	....	....	....	4000 feet	....
Sum = ...	88,000 feet	80,250 feet	80,000 feet	101,750 feet	91,500 feet
H_2 = ...	16·667 miles	15·199 miles	15·152 miles	19·271 miles	17·329 miles
$H_2 \sin\frac{1}{2}\beta$ = ...	3·383 miles	3·933 miles	3·981 miles	4·986 miles	3·008 miles
$H_2 \sin\frac{1}{2}\beta \cos \text{Az.}$ =	3·009 miles	3·933 miles	3·395 miles	2·493 miles	0·235 mile
$H_2 \sin\frac{1}{2}\beta \sin \text{Az.}$ =	−1·536 mile	0·000 mile	1·960 mile	4·318 miles	2·989 miles
E = ...	0·03633 mile	0·06577 mile	0·07217 mile	0·04901 mile	0·00458 mile

TABLE III.—Station B. (Kalianpur), *Known Region.*

Heights of the masses in the compartments above A.	Lune I.	Lune II.	Lune III.	Lune IV.	Lune V.
h_1	....	....	....	....	....
h_2	....	....	....	....	....
h_3	....	....	....	....	....
h_4	....	....	....	....	....
h_5	....	....	....	....	....
h_6	....	....	....	....	....
h_7	....	....	....	....	....
h_8	....	....	....	....	....
h_9	....	....	....	....	....
h_{10}	....	....	....	....	....
h_{11}	....	....	....	....	....
h_{12}	....	....	....	....	....
h_{13}	....	....	....	....	....
h_{14}	....	....	....	....	....
h_{15}	....	....	....	....	....
h_{16}	....	....	....	....	....
h_{17}	....	....	....	....	....
h_{18}	....	....	....	....	....
h_{19}	....	....	....	....	....
h_{20}	....	....	....	....	....
h_{21}	....	....	....	....	....
h_{22}	....	....	500 feet	....	....
h_{23}	....	500 feet	2500 feet	....	....
h_{24}	....	2500 feet	7000 feet	....	....
h_{25}	....	4500 feet	9000 feet	....	....
h_{26}	....	6500 feet	9500 feet	....	....
h_{27}	1200 feet	8200 feet	....	....	....
h_{28}	....	9500 feet	....	....	....
Sum = ...	1200 feet	31,700 feet	28,500 feet	....	....
H_1=	0·227 mile	6·186 miles	5·398 miles	....	....
$H_1 \sin \frac{1}{2}\beta$=	0·016 mile	1·600 mile	1·387 mile	....	....
$H_1 \sin \frac{1}{2}\beta \cos$ Az. =	0·015 mile	1·600 mile	1·211 mile	....	....
$H_1 \sin \frac{1}{2}\beta \sin$ Az. =	—0·005 mile	0·000 mile	0·698 mile	....	....
E=....................	0·00125 mile	0·02941 mile	0·02123 mile	....	....

TABLE IV.—Station B. (Kalianpur), *Doubtful Region.*

Heights of the masses in the compartments above A.	Lune I.	Lune II.	Lune III.	Lune IV.	Lune V.
h_{27}	....	....	Eleven heights, from 9000 feet to 1500 feet	Four heights, from 4000 feet to 9000 feet	....
h_{28}	6000 feet	....			....
h_{29}	9000 feet	Nine heights, from 9000 feet to 2500 feet			....
h_{30}	Five heights, from 9000 feet to 6700 feet			Eight heights, from 9000 feet to 3000 feet	....
h_{31}					....
h_{32}					Three heights, from 5000 feet to 9000 feet
h_{33}					
h_{34}					
h_{35}	....				8250 feet
h_{36}	....				7500 feet
h_{37}	....				....
h_{38}	....	....	2000 feet	....	....
h_{39}	....	....	3000 feet		....
Sum=	54,250 feet	51,750 feet	62,750 feet	74,000 feet	36,750 feet
$H_2=$	10·275 miles	9·801 miles	11·794 miles	14·015 miles	6·960 miles
$H_2 \sin \frac{1}{2}\beta=$	0·716 mile	2·536 miles	3·052 miles	3·626 miles	0·060 mile
$H_2 \sin \frac{1}{2}\beta \cos \text{Az.} =$	0·677 mile	2·536 miles	2·643 miles	1·813 mile	0·011 mile
$H_2 \sin \frac{1}{2}\beta \sin \text{Az.} =$	− 0·238 mile	0·000 mile	1·526 mile	3·141 miles	0·059 mile
E=	0·00874 mile	0·04412 mile	0·05519 mile	0·02941 mile	0·00143 mile

TABLE V.—Station C. (Damargida), *Known Region.*

Heights of the masses in the compartments above A.	Lune I.	Lune II.	Lune III.
h_1	....	....	....
h_2	....	....	....
h_3	....	....	....
h_4	....	....	....
h_5	....	....	....
h_6	....	....	....
h_7	....	....	....
h_8	....	....	....
h_9	....	....	....
h_{10}	....	....	....
h_{11}	....	....	....
h_{12}	....	....	....
h_{13}	....	....	....
h_{14}	....	....	....
h_{15}	....	....	....
h_{16}	....	....	....
h_{17}	....	....	....
h_{18}	....	....	....
h_{19}	....	....	....
h_{20}	....	....	....
h_{21}	....	....	....
h_{22}	....	....	....
h_{23}	....	....	....
h_{24}	....	....	....
h_{25}	....	....	....
h_{26}	....	....	....
h_{27}	....	....	....
h_{28}	....	....	....
h_{29}	....	....	....
h_{30}	600 feet	....	....
h_{31}	4000 feet	....	....
h_{32}	7300 feet	....	....
h_{33}	9500 feet	....	....
Sum = ...	21,400 feet	....	....
H_1=	4·530 miles	....	....
$H_1 \sin\frac{1}{2}\beta$=	1·173 mile	....	....
$H_1 \sin\frac{1}{2}\beta \cos Az.$= ...	1·173 mile	....	....
$H_1 \sin\frac{1}{2}\beta \sin Az.$= ...	0·000 mile	....	....
E=.......................	0·01961 mile	....	....

Table VI.—Station C. (Damargida), *Doubtful Region.*

Heights of the masses in the compartments above A.	Lune I.	Lune II.	Lune III.
h_{31}	....	6000 feet	....
h_{32}	....	Eight heights, from 9000 feet to 1500 feet	....
h_{33}	....		7000 feet
h_{34}	Seven heights, from 9000 feet to 2500 feet		Five heights, from 9000 feet to 5500 feet
h_{35}			
h_{36}			
h_{37}			
h_{38}			
h_{39}			....
h_{40}		2000 feet	....
Sum = ...	40,250 feet	50,000 feet	43,250 feet
H_2=	7·623 miles	9·470 miles	8·192 miles
$H_2 \sin\frac{1}{2}\beta$=	1·973 mile	2·451 miles	1·422 mile
$H_2 \sin\frac{1}{2}\beta \cos$ Az.= ...	1·973 mile	2·103 miles	0·816 mile
$H_2 \sin\frac{1}{2}\beta \sin$ Az.= ...	0·000 mile	1·225 mile	1·165 mile
E=	0·03432 mile	0·04220 mile	0·01143 mile

42. The following results are gathered from these Tables:—

			Known Region.	Doubtful Region.
Station A.	$H \sin\frac{1}{2}\beta \cos$ Az.,	Lune I.	0·772	3·009
	,,	Lune II.	3·656	3·933
	,,	Lune III.	4·262	3·395
	,,	Lune IV.	2·602	2·493
	,,	Lune V.	0·095	0·235
		Totals	11·387	13·065
	E	Lune I.	0·05614	0·03633
	,,	Lune II.	0·08331	0·06577
	,,	Lune III.	0·08066	0·07217
	,,	Lune IV.	0·04901	0·04901
	,,	Lune V.	0·00515	0·00458
		Totals	0·27427	0·22786
	$H \sin\frac{1}{2}\beta \sin$ Az.,	Lune I.	−0·393	−1·536
	,,	Lune II.	0·000	0·000
	,,	Lune III.	2·460	1·960
	,,	Lune IV.	4·223	4·318
	,,	Lune V.	1·057	2·989
		Totals	7·347	7·731

Multiplying these several totals by 1″·1392 (see formula 6. in art. 22.), we have the following results:—

Deflection of plumb-line in meridian	12″·972	14″·881
Correction of same for every 100 feet of change in heights ...	0·312	0·260
Deflection of plumb-line in prime vertical	8·136	8·806

			Known Region.	Doubtful Region.
STATION B.	$K \sin \frac{1}{2}\beta \cos$ Az.,	Lune I.	0·015	0·677
	„	Lune II.	1·600	2·536
	„	Lune III.	1·211	2·643
	„	Lune IV.	0·000	1·813
	„	Lune V.	0·000	0·011
		Totals	2·826	7·680
	E	Lune I.	0·00125	0·00874
	„	Lune II.	0·02941	0·04412
	„	Lune III.	0·02123	0·05519
	„	Lune IV.	0·00000	0·02941
	„	Lune V.	0·00000	0·00143
		Totals	0·05189	0·13889
	$H \sin \frac{1}{2}\beta \sin$ Az.,	Lune I.	−0·005	−0·238
	„	Lune II.	0·000	0·000
	„	Lune III.	0·698	1·526
	„	Lune IV.	0·000	3·141
	„	Lune V.	0·000	0·059
		Totals	0·693	3·488

Multiplying the several totals by $1''\cdot1392$ as before, we have—

Deflection of the plumb-line in meridian	$3''\cdot219$	$8''\cdot749$
Correction of the same for every 100 feet of change in heights...	0·059	0·158
Deflection of the plumb-line in prime vertical	0·789	3·974

STATION C.	$H \sin \frac{1}{2}\beta \cos$ Az.,	Lune I.	1·173	1·973
	„	Lune II.	0·000	2·103
	„	Lune III.	0·000	0·816
		Totals	1·173	4·892
	E	Lune I.	0·01961	0·03432
	„	Lune II.	0·00000	0·04220
	„	Lune III.	0·00000	0·01143
		Totals	0·01961	0·08795
	$H \sin \frac{1}{2}\beta \sin$ Az.,	Lune I.	0·000	0·000
	„	Lune II.	0·000	1·225
	„	Lune III.	0·000	1·165
		Totals	0·000	2·390

Multiplying the several totals by $1''\cdot1392$, we have—

Deflection of plumb-line in meridian	$1''\cdot336$	$5''\cdot573$
Correction of same for every 100 feet of change in heights ...	0·022	0·100
Deflection of plumb-line in prime vertical	0·000	2·723

43. Adding together the results of the last article, we have—

Deflection of plumb-line in meridian at A . . . $=27''\cdot853$
Deflection of plumb-line in meridian at B . . . $=11\cdot968$
Deflection of plumb-line in meridian at C . . . $=\ 6\cdot909$
Deflection of plumb-line in prime vertical at A$=16\cdot942$
Deflection of plumb-line in prime vertical at B$=\ 4\cdot763$
Deflection of plumb-line in prime vertical at C$=\ 2\cdot723$

∴ Difference of meridian deflections at A and B$=15\cdot885$
Difference of meridian deflections at A and C$=20\cdot944$
Difference of meridian deflections at B and C$=\ 5\cdot059$

The first of these quantities is considerably greater than $5''\cdot236$, the quantity brought to light by the Indian Survey. And the values of the deflections at B and C bear a far higher ratio to those at A than has been generally supposed. For even Kaliana was selected by Colonel Everest in the expectation that it would be beyond the sensible influence of mountain attraction; whereas even at C the deflection in the meridian$=6''\cdot909$, if the heights have been rightly assigned in this approximation. In the following articles I shall examine these values more minutely, and consider the effect of various hypotheses for reducing them. In the mean time I will write down the following results from these values for the deflections:—

Total deflection at A$=32\cdot601$, and in azimuth . . . . $31^\circ\ 18'$ East
Total deflection at B$=12\cdot880$, and in azimuth . . . . 21 42 East
Total deflection at C$=\ 7\cdot426$, and in azimuth . . . . 21 31 East*

44. In attempting to reduce the amount of deflection deduced by these calculations, the first thought that comes to mind is, that the density of the attracting mass may have been chosen too large. I have made it 2·75 of distilled water, which is that which was assigned as the mean density of the mountain Schehallien in the calculations of Maskelyne. This can hardly be too great; for, as I shall soon show, a very large share of the deflection is produced by the attraction of the elevated plateau which lies in Thibet and south of that country; and as this is on an average 10,000 feet or more high, the lower part of the materials must be denser rather than lighter than those of a mountain of inconsiderable altitude. If, however, we do reduce the density, say to 2·25, which is yielding much, still the deflections and their differences are reduced by only one-fifth part, and therefore this will not solve the difficulty.

45. The next thought is, that I may have assigned too great a mass to the Doubt-

* Some idea may be formed of the amount of these deflections from the following representation. Conceive three hemispherical mountains of granite to exist close to the three stations A, B, C, their bases being horizontal and just coming up to the stations, and the centres of the bases bearing respectively 31° 18′, 21° 42′, 21° 31′ east of the north meridian. That the horizontal attraction of these hemispherical mountains on the plumb-line at the three stations may be equal to the attraction of the Himalayas and the regions beyond, the diameters of their bases must be respectively 5, 2, $1\frac{1}{8}$ miles very nearly.

ful Region. This I will now examine. All the results which depend upon this part I will multiply by a factor $1-x$. If $x=0$, the results will stand as they do now. If $x=1$, this will amount to supposing the mass standing on the whole Doubtful Region to be non-existent, an hypothesis clearly impossible. By giving x any intermediate fractional value we shall be supposing that all the heights, and therefore the whole mass, are reduced in that ratio. Let A, B, C represent the deflections in meridian at the three stations A, B, and C. Then from art. 42. we gather—

$$\begin{aligned} A &= 12''\cdot972+(1-x)\,14''\cdot881 \\ &= 27\cdot853-14\cdot881\,x \\ B &= 3\cdot219+(1-x)\,8\cdot749 \\ &= 11\cdot968-8\cdot749\,x \\ C &= 1\cdot336+(1-x)\,5\cdot573 \\ &= 6\cdot909-5\cdot573\,x\,; \\ \therefore\quad A-B &= 15\cdot885-6\cdot132\,x \\ A-C &= 20\cdot944-9\cdot308\,x \\ B-C &= 5\cdot059-3\cdot176\,x. \end{aligned}$$

These show that the extravagant hypothesis of supposing $x=1$, or that the whole mass on what we have called the Doubtful Region is non-existent, will not reduce the difference of deflections at A and B lower than $9''\cdot753$, which is greater than $5''\cdot236$ in the ratio of 13 : 7. Nor will this even come down sufficiently if we reduce also the density of the remaining mass, that on the Known Region.

46. A third means of reduction may be looked for in the Known Region. By examining the results gathered together in art. 42, it will be seen that the chief part of the meridian attraction of the mass on the Known Region arises from the lunes II. III. and IV. for A, and lunes II. and III. for B. By attentively examining these columns in Tables I. and III. in art. 41, we see that a large portion of the attraction arises from the Great Plateau. The result I arrive at is, that of the deflection $12''\cdot972$ at A, as much as $8''\cdot772$ arises from this plateau; and of the $3''\cdot219$ at B, as much as $2''\cdot010$ arises from the same cause. Hence if $1-y$ be an arbitrary factor,

$$\begin{aligned} &A\ \text{(Known Region)} = 4''\cdot200+(1-y)\,8''\cdot772 \\ &B\ \text{(Known Region)} = 1\cdot209+(1-y)\,2\cdot010 \\ &A-B\ \text{(Known Region)} = 2\cdot991+(1-y)\,6\cdot762. \end{aligned}$$

It will be necessary, then, to cut down the height of the plateau as much as 6000 feet, to make this come down to $5''\cdot236$; or, if we suppose that all the heights in the other part of the Known Region are twice too large, and if we therefore replace $2''\cdot991$ by its half, $1''\cdot496$, even then y must equal 0·55, and the elevation of the plateau* above the sea be reduced from 10,000 feet to 6000 feet. And all this *in addition to* the hypothesis of the non-existence of the whole mass on the Doubtful Region!

* I should mention to what extent I assume that this plateau is comprised within what I have called the Known Region. Let four points be marked down on the map, viz. W in lat. 34° and long. 76°, X in lat. 32° 45′

It appears, in short, to be quite hopeless by any admissible hypothesis to reduce the calculated deflection so as to make it tally with the error brought to light by the Survey. In the conclusion of this paper, however, it will appear that such a reduction is not necessary for reconciling the discrepancy.

47. I will here write down the formulæ in their most general shape. Let $1-z$ be a factor (similar to $1-x$ and $1-y$) for the part of the Known Region not including the Plateau. Suppose also the whole of the heights of the Known Region are reduced by a hundreds of feet, and those of the Doubtful Region by b hundreds (in this way I bring in the correction E at the foot of the six Tables). Then

$$A=(1-z)4''\cdot200+(1-y)8''\cdot772+(1-x)14''\cdot881-0''\cdot312a-0''\cdot260b.$$

$$B=(1-z)1''\cdot209+(1-y)2''\cdot010+(1-x)\ 8''\cdot749-0''\cdot059a-0''\cdot158b.$$

$$C=(1-z)0''\cdot729+(1-y)0''\cdot607+(1-x)\ 5''\cdot573-0''\cdot022a-0''\cdot100b.$$

The method of using these arbitrary symbols is this. If it appear on examining the contour of the earth's surface more carefully, that the heights in the Known Region and south of the space I call the Plateau, ought to be reduced in a certain ratio, we have but to give the value of that ratio to z: the same is the case, as I have already shown, with y and the Plateau itself. If, on the other hand, we do not wish to reduce the heights in the Known Region in a certain ratio, but by the same given quantity, z and y must be put $=0$, and $a=$ the number of hundreds of feet by which we wish to reduce. Thus if we wish to reduce the whole Known Region by 1000 feet in altitude, we must put $a=10$. We may, moreover, combine these methods of reduction, and both reduce the general ratio of the heights, and afterwards cut these reduced heights down by a constant quantity by giving z, y, and a the proper values. The same things may be done for the Doubtful Region by assigning proper values to x and b: so that the formulæ here given admits of adaptation to various hypotheses of reduction.

48. We may use these last formulæ for comparing the masses which stand on the Known and Doubtful Regions with each other, and with the mass of the earth. In doing this I shall suppose that the heights have been rightly assigned in the present paper. Therefore $z=0$, $y=0$, $x=0$. Then if $a=41\cdot58$, the part of A which arises from the Known Region is reduced to zero. Hence 4158 feet is the average height of the mass standing on that part. In the same manner, by making $b=57\cdot30$, the part of A which arises from the Doubtful Region vanishes; and therefore the average height of the whole mass standing on that portion of the Enclosed Space is 5730 feet. It will be observed that this is greater than the former. The obvious reason of this is, that the mass of the Known Region is highest at its furthest parts from A, whereas the reverse is the case with the mass of the Doubtful Region. The superficial extent of each

and long. 76° 45′, Y in lat. 30° 30′ and long. 80°, and Z in lat. 31° 30′ and long. 83°. Join W and Z by a circle about Kaliana as centre, and join these and the other points by arcs of great circles of the sphere. I take the average height of the enclosed mass to be 10,000 feet above the sea, a mean altitude which I conceive is rather under than over the mark.

of these two regions is found by the following formula of spherical trigonometry:—
Area of portion of a lune

$$=\frac{\pi\beta r^2}{180}(\sin\alpha_n-\sin\alpha_m),$$

the portion being bounded by arcs of great circles at distances α_m and α_n from A, β the angle of the lune expressed in degrees, and r being the radius of the earth. The values of α_m and α_n for the several lunes are obtained from arts. 40. and 25. The results are, that the superficial extents of the Known and Doubtful Regions equal respectively $0{\cdot}1679616\,r^2$ and $0{\cdot}7555430\,r^2$ square miles, $r=4000$; and therefore putting the density = half that of the earth's mean density, and the respective heights $=4158$ feet or 0·7761 mile, and 5730 feet or 1·0852 mile, we have the following results:—

$$\frac{\text{mass on Known Region}}{\text{mass of the earth}}=\frac{3}{8\pi}\,\frac{0{\cdot}1679616\times 0{\cdot}7761}{r}$$

$$=0{\cdot}000003890$$

$$\frac{\text{mass on Doubtful Region}}{\text{mass of the earth}}=\frac{3}{8\pi}\,\frac{0{\cdot}7555430\times 1{\cdot}0852}{r}$$

$$=0{\cdot}000024367.$$

Hence, also, $$\frac{\text{mass on Doubtful Region}}{\text{mass on Known Region}}=6{\cdot}264,$$

or the mass on the Doubtful Region is greater than that on the Known Region in a ratio higher than 25 : 4.

Also $$\frac{\text{mass on whole Enclosed Space}}{\text{mass of the earth}}=0{\cdot}000028257.$$

49. By means of this last result we can determine at what distances from A, B, and C, the whole attracting mass must be imagined concentrated in a point, so as to produce the deflections at those three stations found in art. 43. If, instead of multiplying the totals in art. 42. by $1''{\cdot}1392$, we multiply them by $0{\cdot}000005523\,g$ (see art. 21.), we shall have the attractions in terms of gravity. Hence

attraction at A in meridian $=24{\cdot}452\times 0{\cdot}000005523\,g$;
attraction at A in prime vertical $=15{\cdot}078\times 0{\cdot}000005523\,g$;
∴ total attraction on A . . . $=28{\cdot}72\times 0{\cdot}000005523\,g$
$=0{\cdot}00015862\,g$.

In the same way I find

Total attraction on B . . . $=0{\cdot}00006245\,g$.
Total attraction on C . . . $=0{\cdot}0000360\,g$.

Hence $$\frac{\text{distance from A of point of concentration}}{\text{radius of earth}}$$

$$=\sqrt{\frac{\text{attracting mass}}{\text{mass of earth}}\times\frac{1}{0{\cdot}00015862}}$$

$$=\sqrt{\frac{0{\cdot}000028257}{0{\cdot}00015862}}=\cdot 422;$$

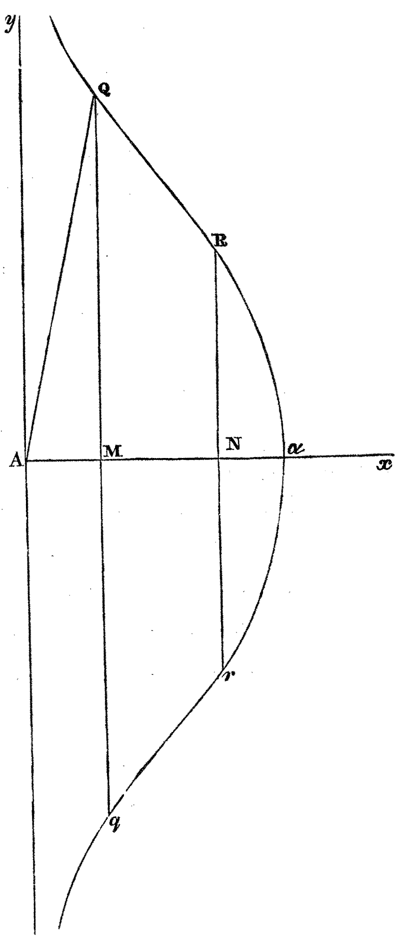
∴ distance from A of point of concentration $=1688$ miles.

In the same manner it may be shown that

distance from B of the point of concentration $=2692$ miles;
distance from C of the point of concentration $=3544$ miles.

The differences of these two last from the first are far greater than the distances of B and C from A, viz. 5° 23′ 37″ and 11° 27′ 33″. From this it is easily inferred, what indeed did not need this proof, that the mass in no sense whatever, even an approximate one, attracts as if concentrated in a fixed point.

Fig. 6.



50. It is extremely difficult to obtain a simple law of attraction, even an approximate one, of such a mass as that under consideration. I have, however, arrived at one which appears to represent the facts with a considerable degree of exactness, and by which we can interpolate for the amount of deflection at any station of the arc intervening between Kaliana and Damargida. It depends upon the properties of the curve of which the equation is $y^2=\frac{a^4}{x^2}-x^2$. Let Ax, Ay be the axes of x and y, and Qaq be the curve; the axis of y is an asymptote, and the curve cuts the axis of x at right angles at a distance A$a=a$. The property of this curve which I am about to use is as follows. The attraction upon A of any slender prism of matter Qq, parallel to the axis of y and terminated at both ends by the curve, is the same as if the mass of the prism were concentrated in the point a. This property is easily demonstrated. For by the process pursued in the proof of the Lemma (art. 13.), it can be shown that the attraction on A of the half prism QM in the direction AM

$$=\frac{\text{mass of QM}}{\text{AM.AQ}};$$

$$\therefore \text{ attraction of Q}q\text{ on A in AM}=\frac{\text{mass of Q}q}{\text{AM.AQ}}$$

$$=\frac{\text{mass of Q}q}{x\sqrt{x^2+y^2}}=\frac{\text{mass of Q}q}{a^2}$$

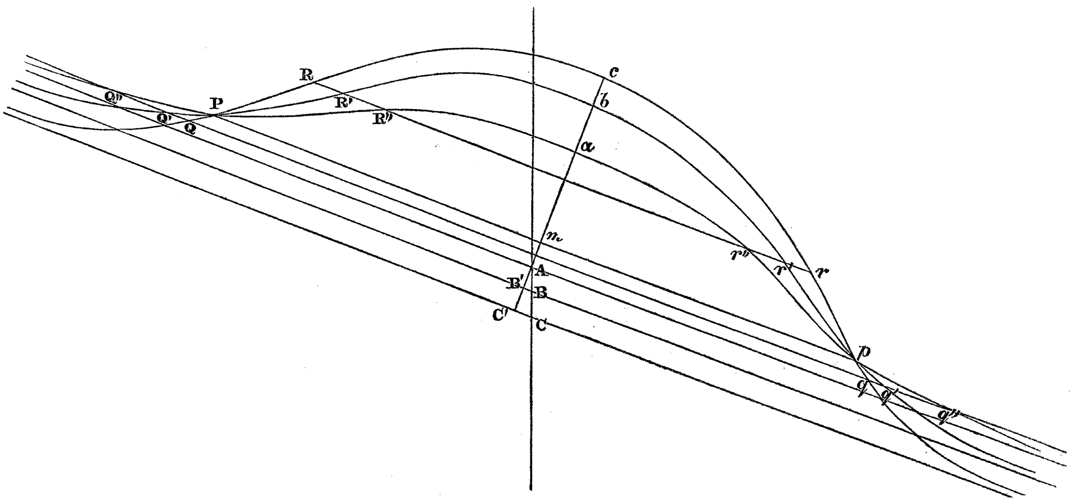
by the equation to the curve. This demonstrates the property. The same, then, is true of any line parallel to y: and it follows easily that a mass of the form QRrq and of uniform thickness will attract the point A as if concentrated in a. But more than this; the property is true also for any prisms parallel to y and lying above the plane xy: so that the mass lying on QRrq need not be of uniform thickness, but may vary in any manner whatever, so long as its height is always small and the same at the same distance from the axis of y.

The property, then, in its most general form is this. If a mass lie on the space Q R rq, always of small comparative altitude and having its section perpendicular to the axis of y always the same (so far as it falls within the curve), the section itself being of any conceivable form whatever, then the attraction of the mass on A is precisely the same as if it were concentrated in the point a (I should mention that in drawing the figure I have for convenience made the dimensions in y one-third of the size they should be compared to those of x).

51. In the previous articles we have seen that by far the largest part of the attraction, at the three stations, arises from the elevated regions lying parallel to the line QR in figs. 4 and 5, running from west-north-west to east-south-east. If, then, we take A in the figure of last article to be Kaliana, and Ax in the north-north-east direction, we may make the parts of QRrq nearest A coincide as nearly as possible with the attracting mass of the Himalayas and the regions beyond, by giving the vertical section MN the right form: and the parts of this mass towards the extremities, and therefore furthest from A, will have a less and less effect in the direction Ax, both on account of their increasing distance and the larger angle at which they act with Ax. Hence we can easily conceive that such a form can be given to the transverse section MN as to make the mass on QRrq *in its effect in the direction* Ax a very fair representative of the actual mass, producing the deflection of the plumb-line at Kaliana. This being the case, the property I have demonstrated, together with the result arrived at in art. 49, shows that Aa, or a in the equation to the curve, $=1688$ miles.

52. By taking different values for a in the curve we may find curves differing in their dimensions, but possessing the same property.

Fig. 7.



In the accompanying figure (fig. 7) three such curves are drawn having a common ordinate Pp, so that the three masses on QRrq, Q$'$R$'r'q'$, and Q$''$R$''r''q''$ are of the same length and coincide precisely with each other, except in a very trifling degree at the two extremities. AB$'$C$'$ are the three attracted points for the three curves: *abc* are the three points where the attracting mass may be conceived to be concentrated, corresponding respectively to AB$'$C$'$. ABC is the meridian through the three stations Kaliana, Kalianpur, and Damargida. It is evident that the attractions of the mass on A, B, and C, in the direction parallel to Aa, will not differ from the attractions on A, B$'$, and C$'$ by any appreciable quantities.

Let A$a=a$, B$'b=b$, C$'c=c$. By art. 49, we have shown that $a=1688$ miles, $b=2692$, $c=3544$. I will now show the consistency of these results, as flowing from the property I have enunciated.

The Trigonometrical Survey shows that AB=371 miles, and BC=430. AB makes $22°\frac{1}{2}$ with AB$'$, and cos 22° 30$'$=0·92388. Hence AB$'$=343 miles, and AC$'$=740. Taking A as the origin of coordinates, and the axes as before, the equations to the three curves give (A$m=x$)

$$\text{P}m^2=\frac{a^4}{x^2}-x^2,\ \text{P}m^2=\frac{b^4}{(x+343)^2}-(x+343)^2$$

and

$$\text{P}m^2=\frac{c^4}{(x+740)^2}-(x+740)^2.$$

If we put $a=1688$ and $b=2692$ in the first and second of these and equate them, the equation is satisfied by $x=222$ miles. And when this value of x is put in the equation formed by equating the first and third values of Pm^2, we obtain

$$c=\sqrt[4]{962^2\left\{\frac{1688^4}{222^2}-222^2+962^2\right\}}$$
$$=3559 \text{ miles.}$$

The very close agreement of this result with that obtained from the calculations of this paper, and shown in art. 49. to be 3544 miles, shows how exactly the law here deduced represents the facts of the case. The value of x, viz. 222 miles, places the line Pp on the part we have called the Plateau, running W.N.W. and E.S.E. about thirty miles north of Gertope.

53. The law thus developed by aid of the curve enables us to interpolate the amount of the deflection of the plumb-line at any station of the arc between Kaliana and Damargida. Thus suppose X is the distance of the station from the fixed line Pp, and A the distance of the centre of concentration for that station. Then

$$\frac{\text{A}^4}{\text{X}^2}-\text{X}^2=\text{P}m^2,\ \text{and this}=\frac{a^4}{222^2}-222^2;$$

$$\therefore\ \frac{\text{A}^4}{a^4}=\frac{\text{X}^2}{222^2}\left\{1+\frac{(\text{X}^2-222^2)222^2}{a^4}\right\}.$$

Since X is never greater than 962, and $a=1688$, the second term within the bracket will always be an extremely small fraction, which may be neglected;

$$\therefore \quad \frac{A^2}{a^2}=\frac{X}{222};$$

$$\therefore \quad \frac{\text{deflection of plumb-line at proposed station}}{\text{deflection of plumb-line at Kaliana}}$$

$$=\frac{\text{attraction at the station}}{\text{attraction at Kaliana}}=\frac{a^2}{A^2}=\frac{222}{X}.$$

But by art. 43. deflection at Kaliana $=32''\,601$;

$$\therefore \quad \text{deflection at the station in question }=\frac{7237''\cdot422}{X}.$$

If we had used the second and third values of Pm^2 above, we should have obtained from the results for Kalianpur and Damargida given in art. 43., the following:—

$$\text{deflection at the station in question }=\frac{7277''\cdot200}{X}$$

$$\text{deflection at the station in question }=\frac{7143''\cdot812}{X}.$$

These three formulæ are very nearly the same. I shall adopt the mean of them; and in taking the mean I shall give the three their respective "weights," which are as the numbers 76, 30, 17, as the deflections in art. 43. show. This leads to the following formula:—

Deflection of plumb-line, at a station the distance of which from Pp is X miles,

$$=\frac{7235''}{X}.$$

54. We may obtain X in terms of the latitude. Let L be the latitude of the place, and l the latitude of Kaliana; then $X=222+(l-L)\cos 22°\,30'\times$ the number of miles in a degree of latitude at the centre of the Indian Arc. The value of the multiplier of $l-L$ which best suits my purpose is 63·06465. As this $=69\cos 23°\,56'$, using this value shows that if the length of a degree in this latitude $=69$ miles, then the line Pp must run about the one-eighth of a point further from east and west than I have placed it, viz. W.N.W. and E.S.E. If the length of the degree be less than sixty-nine miles, then Pp will need a still smaller shift of position.

Substituting the above value, we have deflection at any station of which the latitude is $l-L$ degrees south of Kaliana

$$=\frac{7235''}{63\cdot06465(l-L)+222}=\frac{114''\cdot712}{l-L+3\cdot520}.$$

If we put $l-L=0$, $5°\,23'\,37''$, and $11°\,27'\,33''$ corresponding with the latitudes of Kaliana, Kalianpur, and Damargida, this formula gives for the three deflections at those stations $32''\cdot590$, $12''\cdot870$, and $7''\cdot668$. These so nearly accord with the values in art. 43, that the above formula will represent very approximately the deflection at any station on the whole arc.

55. It may easily be shown from the results of the foregoing articles, that the formula above amounts to supposing, that the Himalayas and the regions beyond attract the three stations and all intermediate stations on the arc between Kaliana and Damargida, as a bar of matter would, running about W.N.W. and E.S.E. at a distance of 222 miles from Kaliana, the length of the bar being infinite, its transverse section small, and its density such that the mass of the Himalayas and attracting regions beyond shall equal the mass of 1284 miles of its length.

56. In order to deduce from this formula for the total deflection the deflection in the meridian, which is the part of most importance, we ought to know the azimuth of the vertical plane in which the total deflection takes place. This is not the same for the three stations A, B, and C. By article 43. it appears that the azimuths at those three stations are 31° 18′, 21° 42′, and 21° 31′. After various trials I find that the following formula represents the law with sufficient exactness. If θ be the azimuth (measured from the north), then

$$\cos\theta=\frac{\cos 31^\circ\, 18'}{1-\frac{1}{10}\sin 10(l-\mathrm{L})}.$$

When $l-\mathrm{L}=0$, 5° 23′ 37″, and 11° 27′ 33″, this gives $\theta=31^\circ\, 18'$, 21° 35′ and 19° 58′. This last differs by 1° 33′ from the value in art. 43; but the second only by 7′. These are sufficiently near the truth, as the cosine in the extreme case will differ from the truth by only about $\frac{1}{100}$th part. The formula for the azimuth departs most from the truth when $l-\mathrm{L}=9^\circ$, that is, at a point about half a degree south of the middle point between Kalianpur and Damargida. But the form of the function is so chosen, that it does not vary much along the whole arc between those stations; and the above-mentioned departure amounts to only one-seventieth part of the proper value. Between Kaliana and Kalianpur I think the formula will represent the azimuth very exactly; and although below that not with the same exactness, yet to a degree of approximation which will introduce no error of importance in the value of the deflection in the meridian.

57. We may show this by combining this formula with that deduced for the total deflection in art. 54. Thus

Deflection in the meridian at any place of which the latitude is $l=$ total deflection $\times \cos\theta$

$$=\frac{114''\cdot 712}{l-\mathrm{L}+3\cdot 520}\times\frac{\cos 31^\circ\, 18'}{1-\frac{1}{10}\sin 10(l-\mathrm{L})}$$

$$=\frac{98''\cdot 016}{(l-\mathrm{L}+3\cdot 520)\left\{1-\frac{1}{10}\sin 10(l-\mathrm{L})\right\}}.$$

When $l-\mathrm{L}=0$, 5° 23′ 37″, and 11° 27′ 33″, this gives for the meridian deflections 27″·845, 11″·965, and 7″·207. These quantities, as calculated from the attracting mass in art. 43, are $27''\cdot853$, $11''\cdot968$ and $6''\cdot909$, which show how good an approximation the formula of this article gives.

58. Before proceeding to the conclusion of this paper I will gather together the formulæ which I have arrived at.

The deflections of the plumb-line at Kaliana, Kalianpur, and Damargida have been found to be as follows:—

In the meridian	$27''\cdot853$,	$11''\cdot968$,	$6''\cdot909$
In the prime vertical . .	$16''\cdot942$,	$4''\cdot763$,	$2''\cdot723$
Total deflections	$32''\cdot601$,	$12''\cdot880$,	$7''\cdot426$
In azimuths	$31°\ 18'$,	$21°\ 42'$,	$21°\ 31'$.

The general formulæ including these results and the deflections for intermediate stations are,—

$$\text{total deflection} = \frac{114''\cdot712}{l-L+3\cdot520};$$

and the azimuth in which its acts is given by

$$\cos\theta = \frac{\cos 31°\ 18'}{1-\frac{1}{10}\sin 10(l-L)},$$

$$\text{deflection in meridian} = \frac{98''\cdot016}{(l-L+3\cdot520)\left\{1-\frac{1}{10}\sin 10(l-L)\right\}}.$$

The formulæ for altering the deflections in meridian for any change in the heights of the attracting mass are brought together in art. 47. Similar formulæ might easily be calculated for the deflections in the prime vertical. If any change be made in the heights of the attracting mass, these formulæ will show what corrections must be introduced into the expressions for the total deflections and their azimuths given above, and also into the constants in the general formulæ.

Conclusion.

59. Before an arc can be made use of in the problem of the figure of the earth, we must know correctly two things concerning it,—its length and its amplitude. Of the two arcs I have been considering, viz. from Kaliana to Kalianpur, and from Kalianpur to Damargida, the correct lengths are known from the Survey; and, as shown in art. 6, these are altogether unaffected by mountain attraction. The same cannot be said of their amplitudes; and till they can be obtained correctly, the arcs can render no service to the great problem. But the amount of deflection in the plumb-line caused by mountain attraction having been determined, the amplitudes obtained astronomically may be corrected, and the arcs may take their place—and a very important place, owing to their length and the accuracy of the geodetic operations—in the investigation of the earth's form.

60. When the length and amplitude of an arc are known, the formula which I give

in art. 62. establishes a relation between the two quantities on which the figure of the earth, supposed to be a spheroid of revolution, depends, viz. the semi-axis major and the ellipticity. As there are two quantities to be determined, a single arc is not sufficient to enable us to find them, unless the lengths and amplitudes of portions of the arc, as well as of the whole, are known. BESSEL, in a paper which has been translated in vol. ii. of TAYLOR's 'Scientific Memoirs,' has shown from ten arcs measured on various parts of the earth and from portions of five of them, by an application of the principle of least squares, that the mean ellipticity $=\frac{1}{300{\cdot}7}$; and that the semi-axis major $=3271953{\cdot}854$ toises, the length of the toise being to that of the fathom as $1{\cdot}06576542 : 1$. This result coincides almost exactly with the ellipticity which theory assigns, upon the supposition that the earth was once a fluid mass, its strata increasing in density from the surface to the centre, the density of the surface being that of granite, and the mean density being $5\frac{2}{3}$rds that of water—a fact which is generally considered to be a strong argument in favour of the original fluidity of the earth's mass.

61. But by the process described above, the peculiarities of the several arcs are all merged in the mean result. When the calculations for the separate arcs are examined, the values are found to vary on either side of the mean. This variety indicates that the several parts of the earth are not curved precisely according to the same elliptic law. Some may think that this militates against the original fluidity of the earth's mass. I do not think this is a fair inference. If the earth's surface ever were fluid, the science of geology shows us that it must have ceased to be so for many ages: and the interval affords time enough for the operation of that well-established law—that gradual changes of elevation and depression are unceasingly taking place in the surface, arising no doubt from chemical and mineralogical changes in the mass—to modify the original curvature of the various parts, making some greater and others less than before. The argument of the earth's original fluidity lies in the fact, that the *present mean form* is that which the earth must have had when it was fluid.

62. I will conclude this paper by calculating the form of the Indian arc between Kaliana and Damargida and its two subdivisions. The result affords, I think, the only explanation of the discordance between the difference of amplitude as brought out in Colonel EVEREST's work, and by my calculation of the amount of deflection.

The lengths of the three arcs, Kaliana—Kalianpur, Kalianpur—Damargida, and their sum, Kaliana—Damargida (which I shall call arcs I., II., III.), are shown by the Survey to be

$$326859{\cdot}52,\ 367154{\cdot}37, \text{ and } 694013{\cdot}89 \text{ fathoms.}$$

From these the amplitudes may be deduced by assuming a form of the meridian. Colonel EVEREST assumes

$$\text{semi-axis major} = 20922931{\cdot}8 \text{ feet, and ellipticity} = \frac{1}{300{\cdot}8}.$$

The formula for thus calculating the amplitude is derived from the usual relation,

$$\frac{\text{arc}}{\frac{1}{2}\text{axis major}}=(1-e^2)\int_{L}^{l}\frac{dl}{\{1-e^2\sin^2 l\}^{\frac{3}{2}}},$$

where l and L are the latitudes of the extremities of the arc, and e is the eccentricity of the ellipse.

Let a be the semi-axis major;
ε the ellipticity;
λ the amplitude of the arc;
μ the latitude of the middle point of the arc.

Then $e^2=2\varepsilon-\varepsilon^2$, $\lambda=l-L$, $2\mu=l+L$; and the above form leads to the following,

$$\frac{\text{arc}}{a}=\lambda\left\{1-\frac{1}{2}\varepsilon\left(1+3\frac{\sin\lambda}{\lambda}\cos 2\mu\right)-\frac{1}{16}\varepsilon^2\left(1-15\frac{\sin\lambda}{\lambda}\cos\lambda\cos 4\mu\right)\right\};$$

$$\therefore\quad \lambda=\frac{\text{arc}}{a}\left\{1+\frac{1}{2}\varepsilon\left(1+3\frac{\sin\lambda}{\lambda}\cos 2\mu\right)\right.$$
$$\left.+\frac{1}{4}\varepsilon^2\left(\left(1+3\frac{\sin\lambda}{\lambda}\cos 2\mu\right)^2+\frac{1}{4}\left(1-15\frac{\sin\lambda}{\lambda}\cos\lambda\cos 4\mu\right)\right)\right\}.$$

If the values for arcs I., II., III. and a are put in the expression $\frac{\text{arc}}{a}$, we obtain the three first approximate values of the computed amplitudes of the three arcs. They are

0·0937324, 0·1052876, and 0·1990201;

or the same expressed in angles are

5° 22′ 13″·715, 6° 1′ 57″·145, and 11° 24′ 10″·869.

These must be put in the small terms on the right-hand side of the form for λ above, and we have the second approximation to the three values, viz.—

$\lambda'=5° 23' 41''·796$, $\lambda''=6° 3' 53''·286$, $\lambda'''=11° 27' 36''·300$.

A third approximation will not affect these results.

Now the astronomically determined amplitudes are, as I gather from Colonel EVEREST's work *,

5° 23′ 37″·058, 6° 3′ 55″·973, and 11° 27′ 33″·032.

And in art. 43. I have found the differences of deflection of the plumb-line at the extremities of the arcs I., II., III., to be

15″·885, 5″·059, and 20″·944.

* These will be found at pp. lxx, lxxi. Expressed in seconds they are 19417″·058, 21835″·973, and 41253″·031. These are the amplitudes, given under the symbol Δl in the Table of final results at the close of his work, and which form the data for the comparison of the Indian arc with arcs in other parts of the world, for the determination of the mean ellipticity of the earth. It is obvious that these values of Δl are not correct as final data, if mountain attraction is to be taken account of. They should be increased by the differences in deflection in the plumb-line caused by attraction at the extremities of the arcs.

I shall use arbitrary factors with these to give the opportunity of introducing any changes in the deflections that may be thought necessary on a further examination of the contour of the surface of the attracting mass; so that

$$15''\cdot885(1-u),\quad 5''\cdot059(1-v),\quad \text{and } 20''\cdot944(1-w)$$

will be the differences of deflection in the meridian at the extremities of the three arcs I., II., III. As the third of these must be the sum of the other two, we have the relation

$$20\cdot944w=15\cdot885u+5\cdot059v,$$

or

$$w=0\cdot7584u+0\cdot2415v.$$

63. Also, by comparing the above values of the deflections with those in art. 47, we have the relations

$$u=0\cdot1883z+0\cdot4257y+0\cdot3861x,$$
$$v=0\cdot0949z+0\cdot2773y+0\cdot6278x,$$
$$w=0\cdot1657z+0\cdot3898y+0\cdot7044x.$$

64. Before proceeding, I will remark that the amplitude of the arc II. determined astronomically, as given above, is somewhat *greater than* that deduced by calculation from the length of the arc. Unless this can be accounted for by the form of the assumed ellipse, it intimates that there is some disturbing cause north of Damargida which increases the inclination of the plumb-line to that at Kalianpur. Should this be the case, the correction for it may be effected by adding a small quantity to the deflection between Kalianpur and Damargida; that is, by increasing $5''\cdot059(1-v)$, or by diminishing v by some quantity v'.

65. We must now add the meridian deflections to the astronomical amplitudes. The results are the true amplitudes of the three arcs I., II., III., viz.—

$$5^\circ\ 23'\ 52''\cdot943-15''\cdot885u,\quad 6^\circ\ 4'\ 1''\cdot032-5''\cdot059v,\quad \text{and } 11^\circ\ 27'\ 53''\cdot976-20''\cdot944w.$$

A comparison of these with the three values of λ, viz. λ', λ'', λ''' in art. 62. deduced by computation, shows that ε and a have been chosen so as to make λ in each case too small. Let $d\lambda'$, $d\lambda''$, $d\lambda'''$ be the three errors of λ', λ'', λ'''; and suppose $\varepsilon+d\varepsilon$ and $a+da$ are the values of ε and a which will by computation bring out the true amplitudes;

$$\therefore\ \frac{d\lambda'}{\lambda'}=\frac{11''\cdot147-15''\cdot885u}{5^\circ\ 23'\ 41''\cdot796}=0\cdot0005739-0\cdot0008179u,$$

$$\frac{d\lambda''}{\lambda''}=\frac{7''\cdot746-5''\cdot059v}{6^\circ\ 3'\ 53''\cdot286}=0\cdot0003548-0\cdot0002317v,$$

$$\frac{d\lambda'''}{\lambda'''}=\frac{17''\cdot676-20''\cdot944w}{11^\circ\ 27'\ 36''\cdot300}=0\cdot0004284-0\cdot0005076w.$$

By the formula of art. 62. we have

$$\text{arc}=a\lambda'\left\{1-\frac{1}{2}\varepsilon\left(1+3\frac{\sin\lambda'}{\lambda'}\cos 2\mu'\right)\right\}.$$

If we differentiate the logarithm of this, we have

$$0=\frac{da}{a}+\frac{d\lambda'}{\lambda'}-\frac{1}{2}\left(1+3\frac{\sin\lambda'}{\lambda'}\cos 2\mu'\right)d\varepsilon.$$

There will be two similar equations in λ'' and λ'''. Put $\frac{da}{a}=\alpha$, and substitute the value found above, and this becomes, after reduction,

$$0=\alpha+0\cdot0005739-0\cdot0008179u-1\cdot3880d\varepsilon.$$

In a similar manner we obtain the following equations:

$$0=\alpha+0\cdot0003548-0\cdot0002317v-1\cdot6095d\varepsilon,$$
$$0=\alpha+0\cdot0004284-0\cdot0005076w-1\cdot5055d\varepsilon.$$

Eliminating α from the 1st and 2nd, the 1st and 3rd, and the 2nd and 3rd, we have the three following equations in $d\varepsilon$:—

$$0=0\cdot0002191-0\cdot0008179u+0\cdot0002317v+0\cdot2215d\varepsilon,$$
$$0=0\cdot0001455-0\cdot0008179u+0\cdot0005076w+0\cdot1175d\varepsilon,$$

and $$0=0\cdot0000736+0\cdot0002317v-0\cdot0005076w+0\cdot1040d\varepsilon.$$

These are the same as

$$d\varepsilon=-0\cdot000989+0\cdot003693u-0\cdot001046v,$$
$$d\varepsilon=-0\cdot001238+0\cdot006961u-0\cdot004320w,$$
$$=-0\cdot001238+0\cdot003685u-0\cdot001043v,$$ substituting for w by art. 62,

and $$d\varepsilon=-0\cdot000708-0\cdot002228v+0\cdot004881w,$$
$$=-0\cdot000708+0\cdot003702u-0\cdot001049v.$$

The method of least squares shows that the arithmetic mean of these is the nearest value;

$$\therefore\quad d\varepsilon=\tfrac{1}{3}(-0\cdot002935+0\cdot011080u-0\cdot003138v)$$
$$=-0\cdot000978+0\cdot003693u-0\cdot001046v.$$

It is worthy of remark, that the terms depending upon the calculated deflection, viz. those in which u and v enter, are very nearly exactly the same in the mean value and the three separate values of $d\varepsilon$. Adding the mean value of $d\varepsilon$ to ε or $\frac{1}{300\cdot8}$ (which equals 0·003324), we have

$$\text{corrected ellipticity}=0\cdot002346+0\cdot003693u-0\cdot001046v;$$

and by adding the three equations in α together, substituting for $d\varepsilon$ its mean value, and dividing by 3, we have

$$\alpha=-0\cdot0039737-0\cdot0051426u+0\cdot0016881v;$$

$$\therefore\quad \text{corrrected semi-axis major}=a(1+\alpha)$$
$$=a\{0\cdot9960263-0\cdot0051426u+0\cdot0016881v\}.$$

66. With regard to these results, I will first observe, that if mountain attraction be neglected altogether, or $u=1$ and $v=1$,

$$\text{ellipticity}=0\cdot002346+0\cdot003693-0\cdot001046$$
$$=0\cdot005093=\frac{1}{196\cdot3}.$$

This value coincides almost exactly with Colonel EVEREST'S determination of the ellipticity from a comparison of the arcs I. and II. He makes it $\frac{1}{191\cdot 6}$ (see the Table at the end of his work).

67. Let us next see whether, by altering the altitudes of the attracting mass, we can make the curvature of the Indian arc equal to that corresponding with the mean ellipticity. The above form will then give

$$\frac{1}{300\cdot 7} \text{ or } 0\cdot 003324 = 0\cdot 002346 + 0\cdot 003693u - 0\cdot 001046v;$$

$$\therefore\ 3693u - 1046v = 978$$

$$u = \frac{978 + 1046v}{3693} = 0\cdot 265 + 0\cdot 283v.$$

As v cannot be less than 0, u cannot be less than 0·265, or somewhat more than $\frac{1}{4}$, to satisfy this equation. Or the heights laid down in the Tables of art. 41. must be so far cut down as to diminish the difference of deflection at Kaliana and Kalianpur by one-fourth of its amount.

Should it be necessary to allow for the larger astronomical amplitude of the arc II. (noticed in art. 64.), by diminishing v by a quantity v', then

$$u = 0\cdot 265 - 0\cdot 283v' + 0\cdot 283v.$$

The astronomical amplitude of arc II. is $2''\cdot 687$ larger than that obtained by computation from the arc; whereas it should be $1''\cdot 509$* smaller than that determined from computation, if it bear the same relation to the difference of deflections as in the arc I. If then we put

$$v' = \frac{2''\cdot 687 + 1''\cdot 509}{5''\cdot 059} = \frac{4\cdot 196}{5\cdot 059} = 0\cdot 8294$$

$$u = 0\cdot 030 + 0\cdot 283v,$$

and a reduction of the difference of deflections at Kaliana and Kalianpur by something more than $\frac{1}{30}$th part, the difference in arc II. remaining unaltered, will bring the ellipticity out $\frac{1}{300\cdot 7}$. There is, however, no reason for supposing either that the curvature of the Indian arc is precisely equal to the mean curvature of the whole quadrant; nor that the heights of the attracting masses have been made so much too great.

68. If these heights have been rightly assigned in this paper, that is, if $u=0$ and $v=0$, then the ellipticity of the Indian arc

$$= 0\cdot 002346 = \frac{1}{426\cdot 2},$$

which shows that the arc is more curved than it would be if it had the mean ellipticity.

* $15''\cdot 885 : 5''\cdot 059 :: 4''\cdot 738 : 1''\cdot 509$, the quantity used in the text.

The degree of increased curvature may be judged of from this. The height of the middle point of an arc of which the amplitude is λ, above the chord of the arc,

$$=\frac{1}{8}a.\lambda^2\left\{1-\varepsilon\left(\frac{1}{2}+\frac{3}{2}\cos 2\mu\right)\right\}$$

μ being the latitude of the middle point, and λ sufficiently small to allow λ^4 to be neglected. In the arc between Kaliana and Damargida (which is about 800 miles long), $\lambda=0{\cdot}2$, $\cos 2\mu=0{\cdot}67473$, and $a=4000$ miles. Hence height of middle point above the chord $=20(1-1{\cdot}512\varepsilon)$ miles.

For the mean ellipticity, this $=19{\cdot}8992$ miles.

For the ellipticity $\frac{1}{426{\cdot}2}$, this $=19{\cdot}9290$ miles.

For the ellipticity $\frac{1}{196{\cdot}3}$, this $=19{\cdot}8460$ miles.

The ellipticity, therefore, which results from taking account of mountain attraction raises the middle point of the arc by 0·0298 of a mile, or 157 feet; whereas the ellipticity when mountain attraction is neglected depresses the arc through 0·0532 of a mile, or 281 feet. These quantities are nearly in the ratio of 5 : 9. Hence the consideration given to mountain attraction in this paper brings the curvature of the Indian arc nearer to the mean curvature than the neglect of mountain attraction does in the ratio of 5 : 9 *. This is, as far as it goes, in favour of these calculations.

69. The conclusion, then, to which I come is, that there is no way of reconciling the difference between the error in latitude deduced in Colonel Everest's work and the amount I have assigned to deflection of the plumb-line arising from attraction—and which, after careful re-examination, I am decidedly of opinion is not far from the truth, either in defect or in excess—but by supposing, that the ellipticity which Colonel Everest uses in his calculations, although correct as a mean of the whole quadrant, is too large for the Indian arc. This hypothesis appears to account for the difference most satisfactorily. The whole subject, however, deserves careful examination; as no anomaly should, if possible, remain unexplained in a work conducted with such care, labour, and ability, as the measurement of the Indian arc has exhibited.

* An *increased* curvature is, moreover, more in accordance with what might be expected, as the effect of the upheaving of the enormous mass of the Himalayas and neighbouring regions, than a diminished curvature.

Deep River, Cape of Good Hope,
July 12, 1854.